Khovanov Homology and the Involutive Heegaard Floer Homology of Branched Double Covers

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Roadmap

- ☐ Khovanov Homology $Kh(L)$, $\widetilde{Kh}(L)$ [Khovanov]
- ☐ Heegaard Floer Homology $\widehat{HF}(Y^3)$ [Ozsváth-Szabó]
- ☐ Spectral Sequence $\widetilde{Kh}(\bar{L}) \Longrightarrow \widehat{HF}(\Sigma(L))$ [Oz-Sz] = [Ozsváth-Szabó]
- ☐ Involutive HF Homology $\widehat{HFI}(Y^3)$ [Hendricks-Manolescu]
- ☐ Involutive Spectral Sequence [Alishahi-Truong-Z., WIP]
  (see work of Francisco Lin)

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  & Bordered version of the spectral sequence $\widetilde{Kh} \Longrightarrow \widehat{HF}(\Sigma)$
- ☐ Involutive Bordered Floer homology [Hendricks-Lipshitz]

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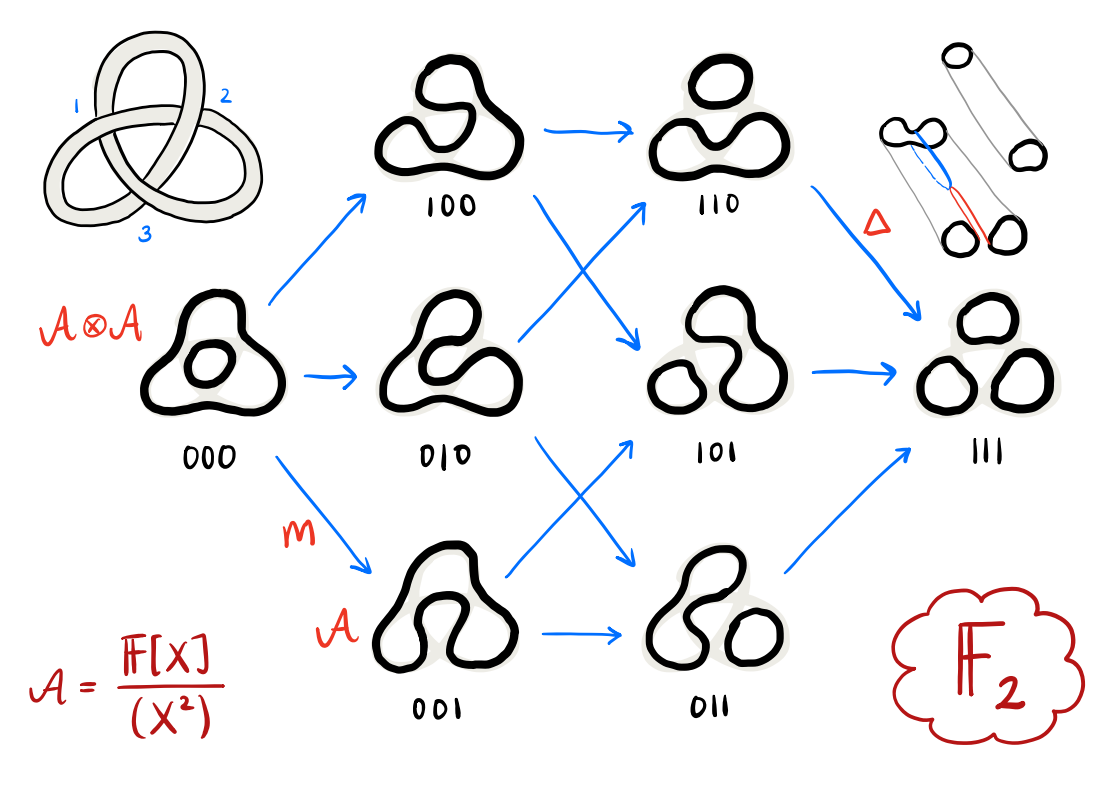
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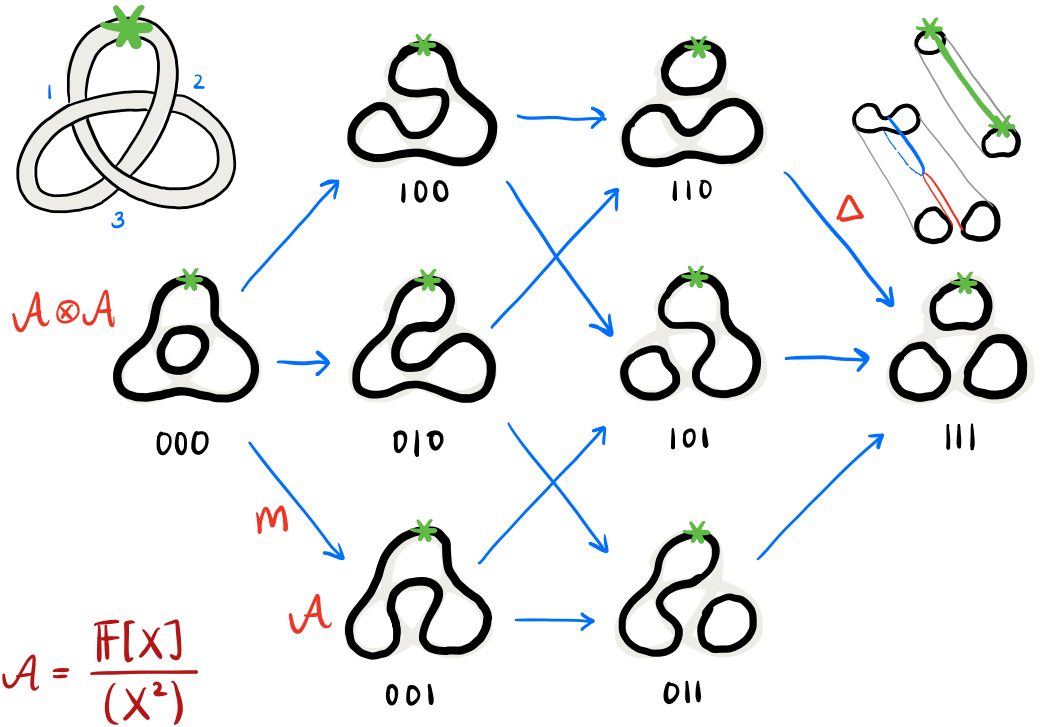
A = F[X] / (X²)
F2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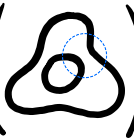
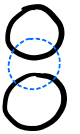
Σ() = Σ()

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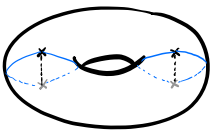
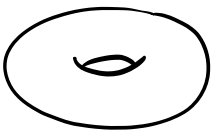
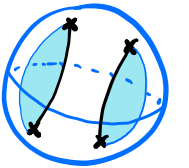
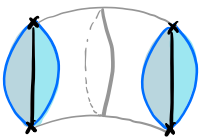
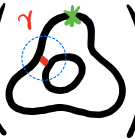
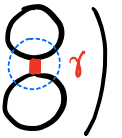
=  $\cup$ 

= $S^1 \times S^2$





$\Sigma($  $) = \Sigma($  $)$

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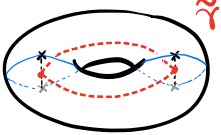
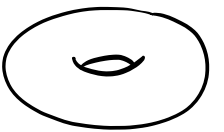
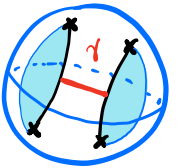
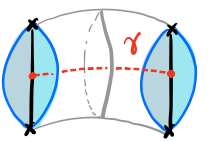
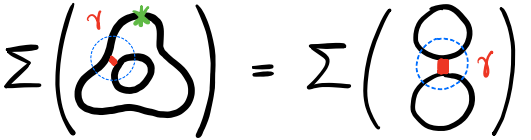
$=$  $\cup$ 

$= (S^1 \times S^2, \tilde{\gamma})$



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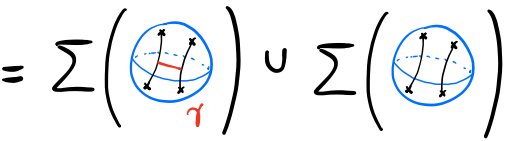
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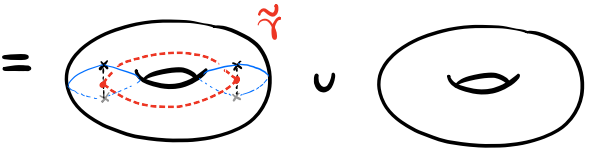
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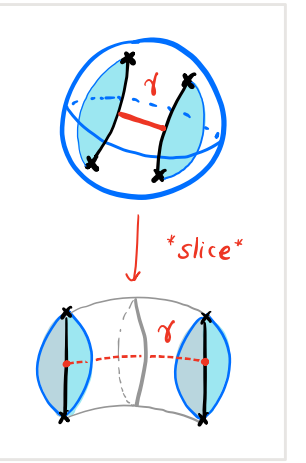
γ
slice
γ


=
(S¹ x S², γ̃)
(-1)-surgery on γ̃
Σ
= S³

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Cone

$$\widehat{CF}(\#^k S^1\times S^2)$$

$$Q\cdot(id+\iota)$$

$$Q\cdot\widehat{CF}(\#^k S^1\times S^2)$$

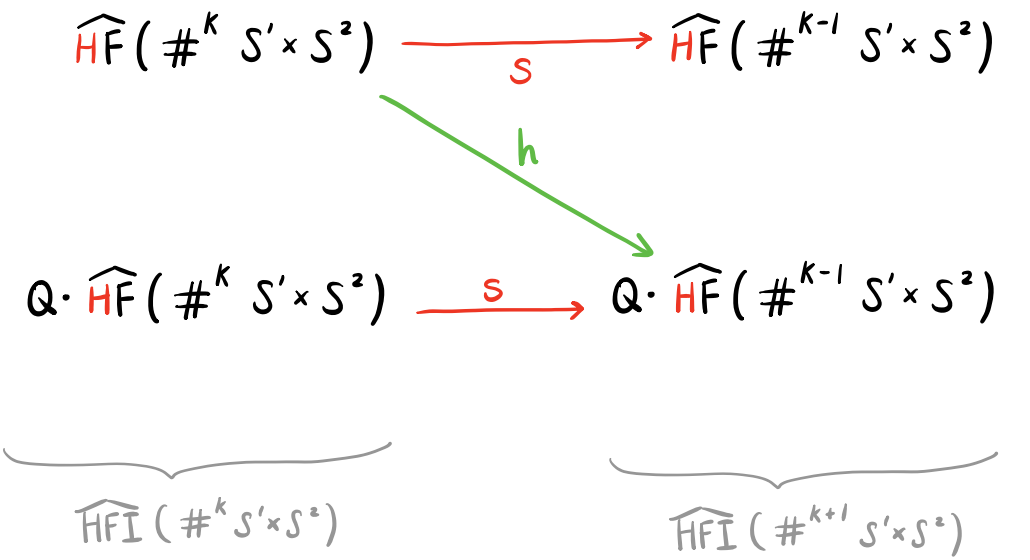
$$= \widehat{CFI}(\#^k S^1\times S^2)$$

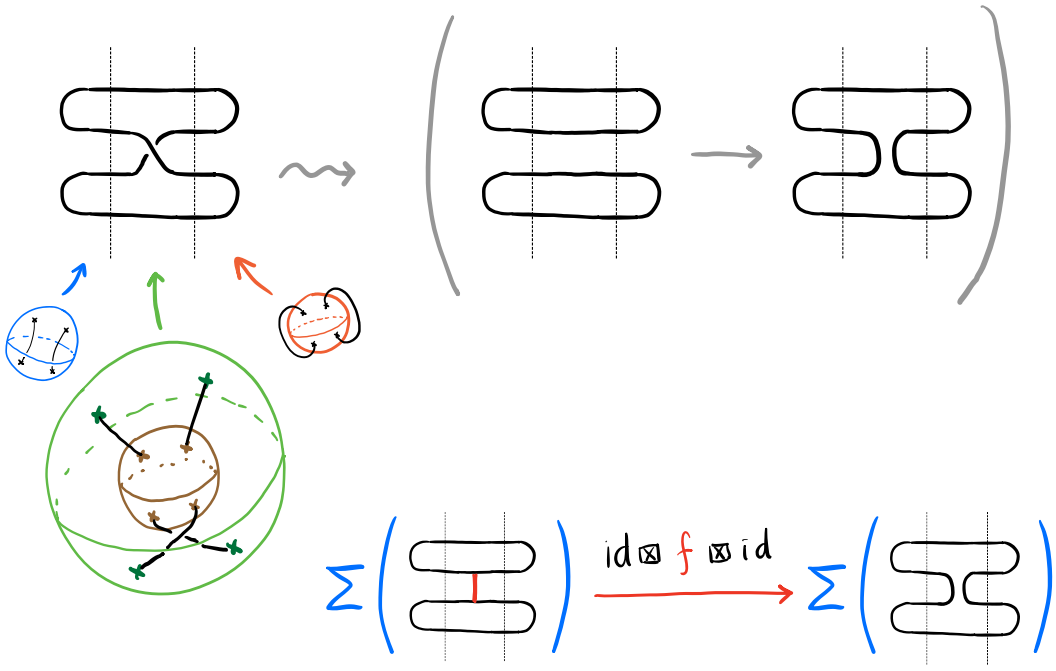
homology

$$\iota \simeq \boxtimes^k \iota_{S^1\times S^2}$$

$$\widehat{HF}(\#^k S^1\times S^2) \oplus Q\cdot\widehat{HF}(\#^k S^1\times S^2)$$

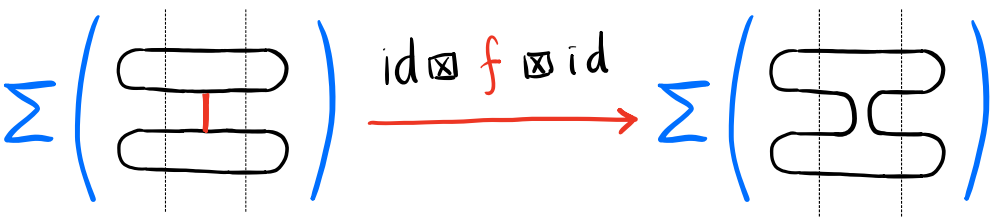
(E', d'):


$\widehat{HF}(\#^k S^1 \times S^2)$
S
$\widehat{HF}(\#^{k-1} S^1 \times S^2)$
h
$Q \cdot \widehat{HF}(\#^k S^1 \times S^2)$
S
$Q \cdot \widehat{HF}(\#^{k-1} S^1 \times S^2)$
$\widehat{HFI}(\#^k S^1 \times S^2)$
$\widehat{HFI}(\#^{k+1} S^1 \times S^2)$

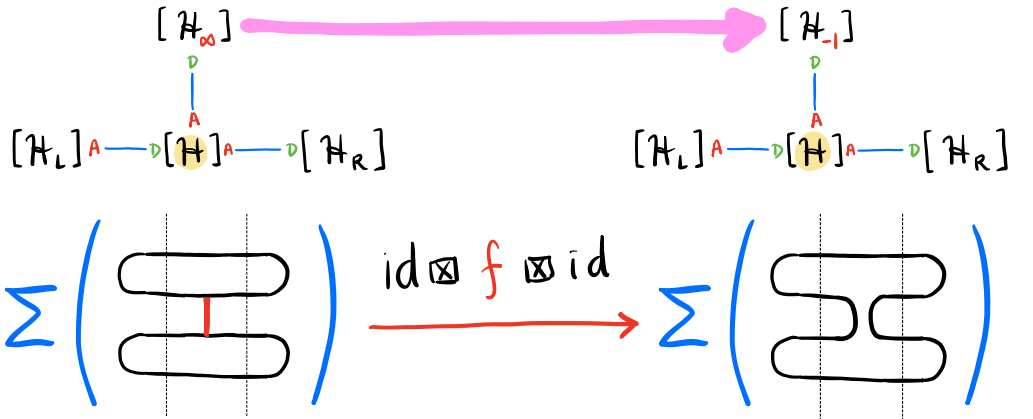

id ⊠ f ⊠ id

$\widehat{CFD}(H_\infty) \boxtimes \widehat{CFA}(\mathcal{H}_L) \boxtimes \widehat{CFDAA}(\mathcal{H}) \boxtimes \widehat{CFD}(\mathcal{H}_R)$

$\widehat{CFD}(H_{-1}) \boxtimes \widehat{CFA}(\mathcal{H}_L) \boxtimes \widehat{CFDAA}(\mathcal{H}) \boxtimes \widehat{CFD}(\mathcal{H}_R)$



$\mathcal{H}$ = bordered diagram for complement of γ



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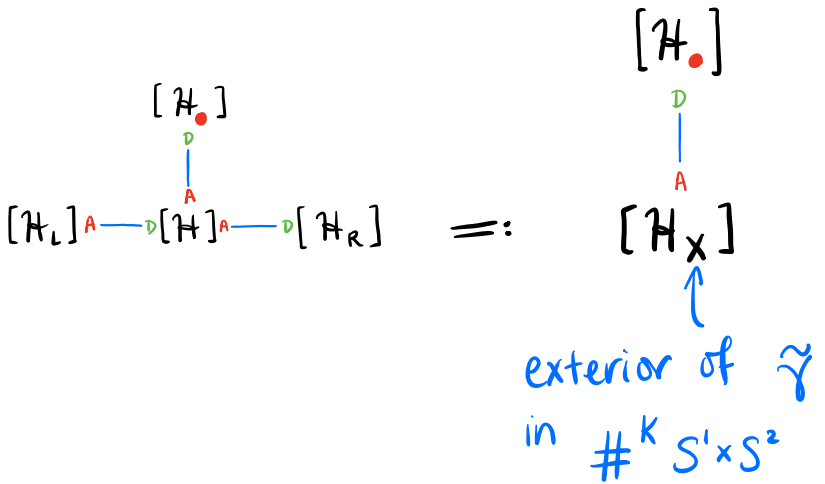
[H•]
D
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[H_L] A — D [H] A — D [H_R]
=:
[H•]
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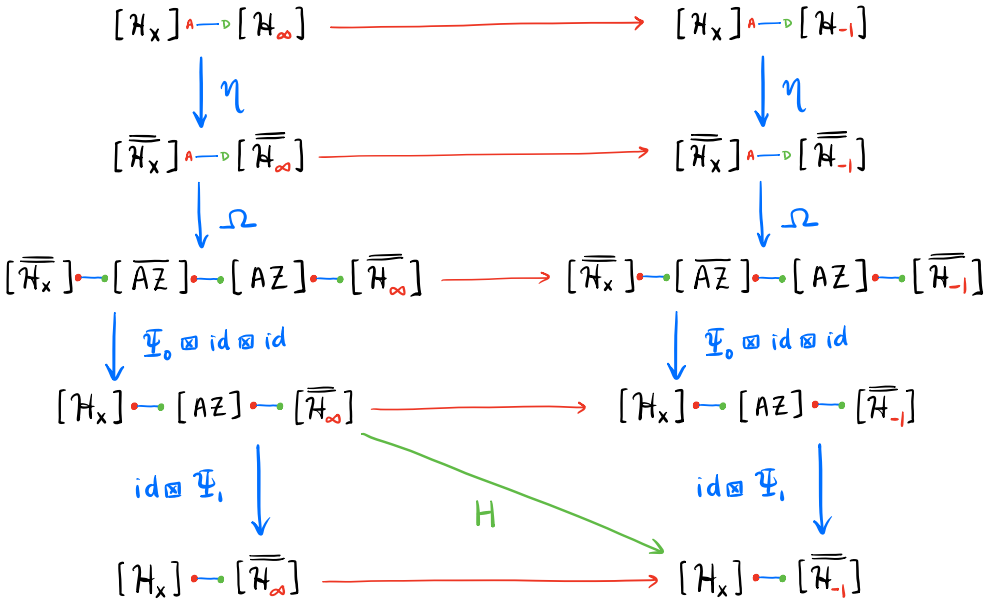
exterior of Ỹ
in #^k S¹×S²


$[\mathcal{H}_X] \to [\mathcal{H}_\infty]$
$[\mathcal{H}_X] \to [\mathcal{H}_{-1}]$
η
$[\overline{\overline{\mathcal{H}_X}}] \to [\overline{\overline{\mathcal{H}_\infty}}]$
$[\overline{\overline{\mathcal{H}_X}}] \to [\overline{\overline{\mathcal{H}_{-1}}}]$
Ω
$[\overline{\overline{\mathcal{H}_X}}] - [\overline{AZ}] - [AZ] - [\overline{\overline{\mathcal{H}_\infty}}]$
$[\overline{\overline{\mathcal{H}_X}}] - [\overline{AZ}] - [AZ] - [\overline{\overline{\mathcal{H}_{-1}}}]$
$\Psi_0 \boxtimes id \boxtimes id$
$[\mathcal{H}_X] - [AZ] - [\overline{\overline{\mathcal{H}_\infty}}]$
$[\mathcal{H}_X] - [AZ] - [\overline{\overline{\mathcal{H}_{-1}}}]$
$id \boxtimes \Psi_1$
H
$[\mathcal{H}_X] - [\overline{\overline{\mathcal{H}_\infty}}]$
$[\mathcal{H}_X] - [\overline{\overline{\mathcal{H}_{-1}}}]$

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