Rank Inequalities for the Knot Floer Homology of (1,1)-Satellites

Weizhe Shen, Georgia Tech

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the Knot Floer Homology
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Satellites

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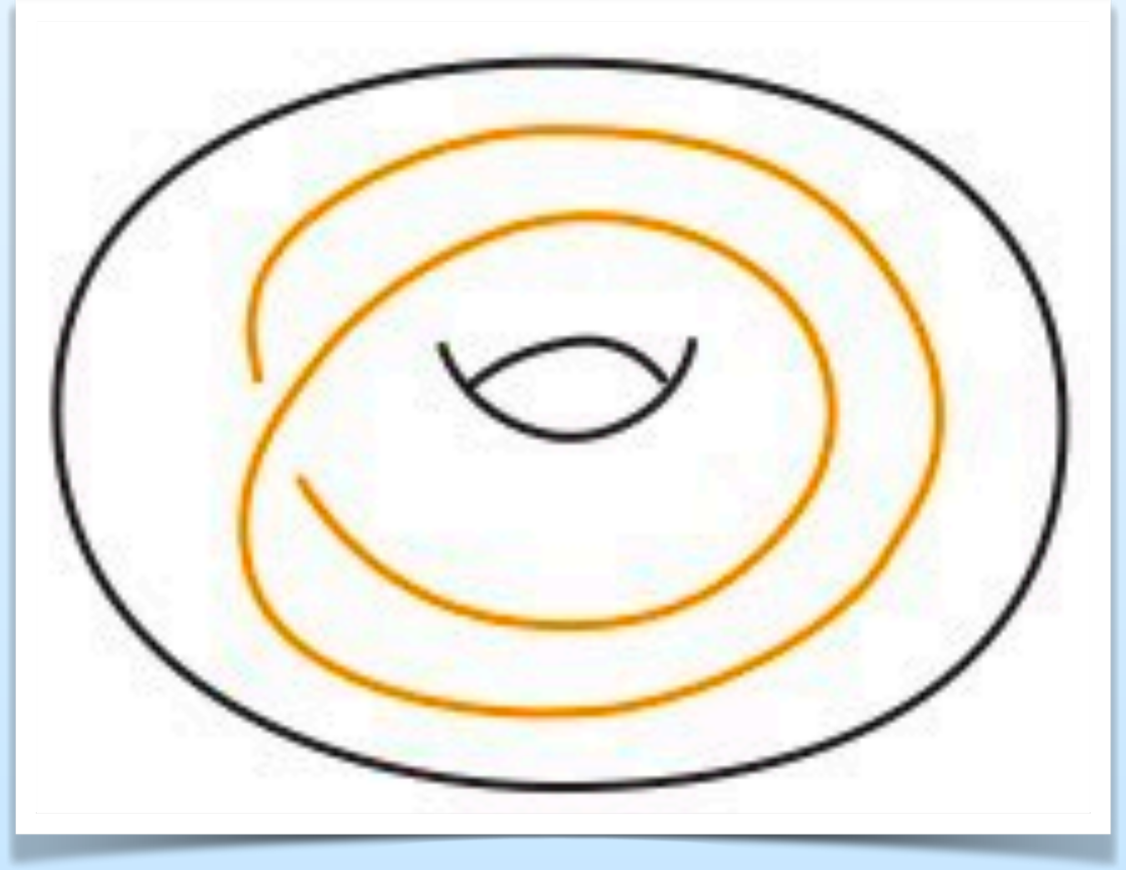

$+$

$=$

$P \subset D^2 \times S^1$

pattern

$K \subset S^3$

companion

$P(K) \subset S^3$

satellite

(1,1)-Satellites

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Definition.

A satellite knot $P(K)$ is called a **(1,1)-satellite** if P is a (1,1)-pattern.

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Examples.

- torus knots
- the Mazur pattern

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Knot Floer Homology
(Ozsváth-Szabó, Rasmussen)

$K \subset S^3$

Knot Floer Homology
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$K \subset S^3$ $\longrightarrow$ $\widehat{HFK}(K)$

a finitely generated
abelian group

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Rank Inequalities

Theorem (Shen).

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- rank $\widehat{HFK}(P(K)) \geqslant$ rank $\widehat{HFK}(K)$

Idea of Proof

Theorem (W. Chen)

Theorem by example: $(T_{2,3})_{3,1}$

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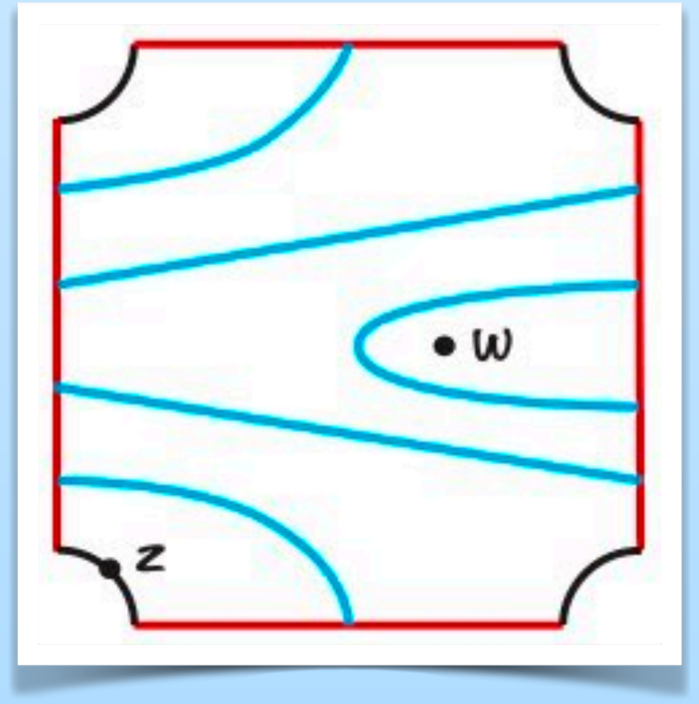
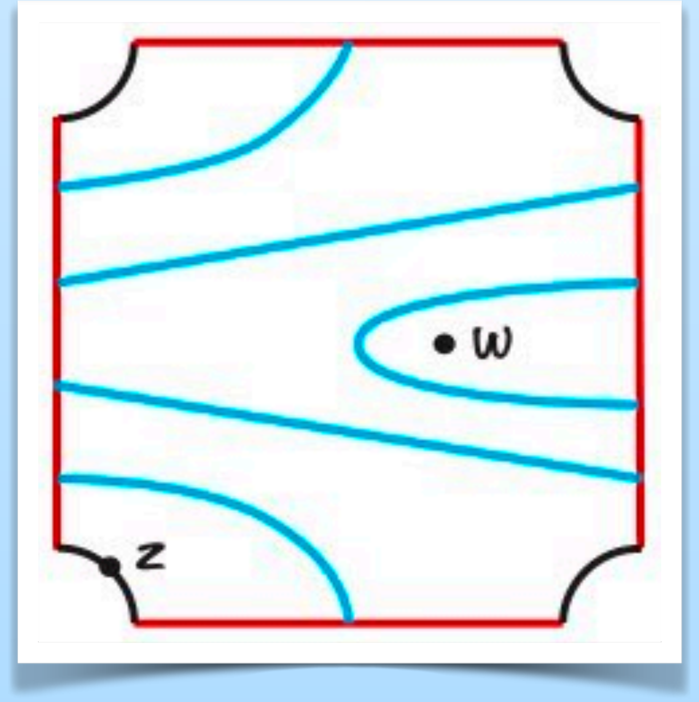
a genus one doubly pointed bordered Heegaard diagram for $(D^2 \times S^1, P)$



Theorem by example: $(T_{2,3})_{3,1}$

a genus one doubly pointed bordered Heegaard diagram for $(D^2 \times S^1, P)$

immersed curves for K in the punctured torus



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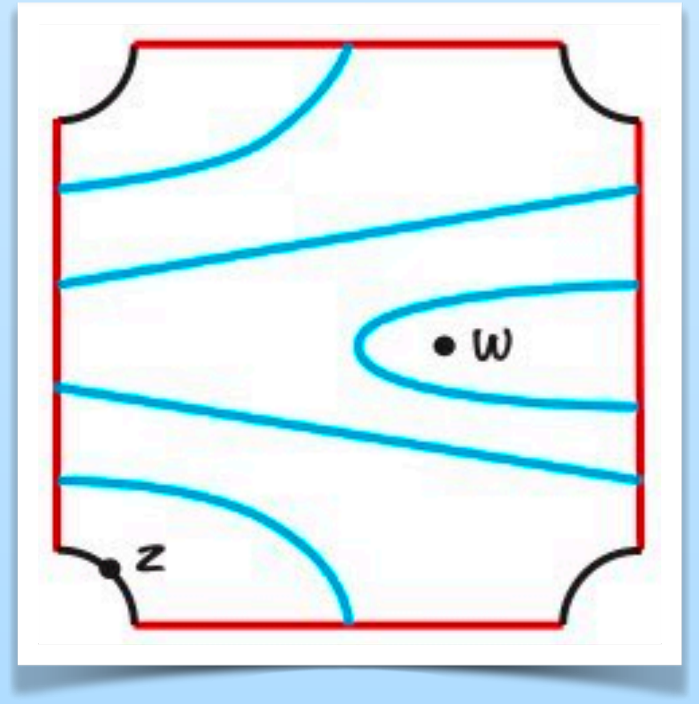

Immersed Curves
(Hanselman-Rasmussen-Watson)

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$$X_K = \overline{S^3 - \nu(K)}$$

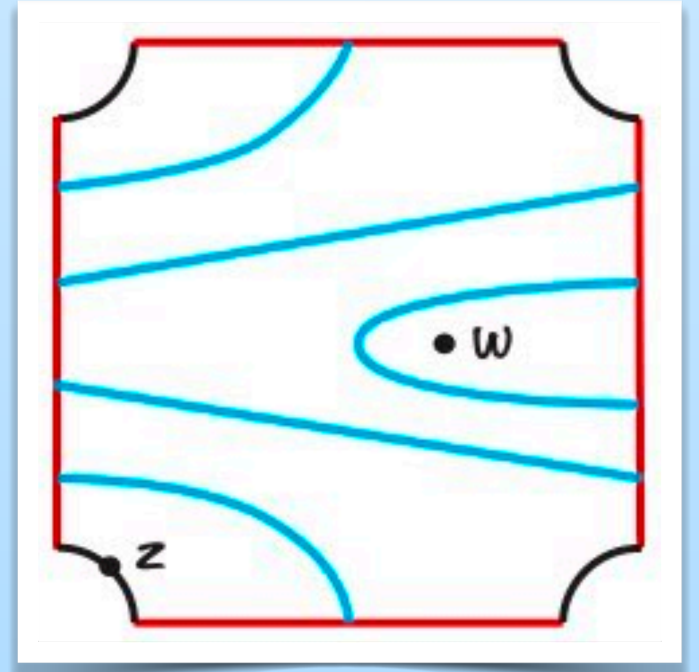
Immersed Curves
(Hanselman-Rasmussen-Watson)

$$X_K = \overline{S^3 - \nu(K)} \longrightarrow$$

a collection of immersed curves in $\partial X_K - \{\text{pt}\}$ (each decorated with a local system)

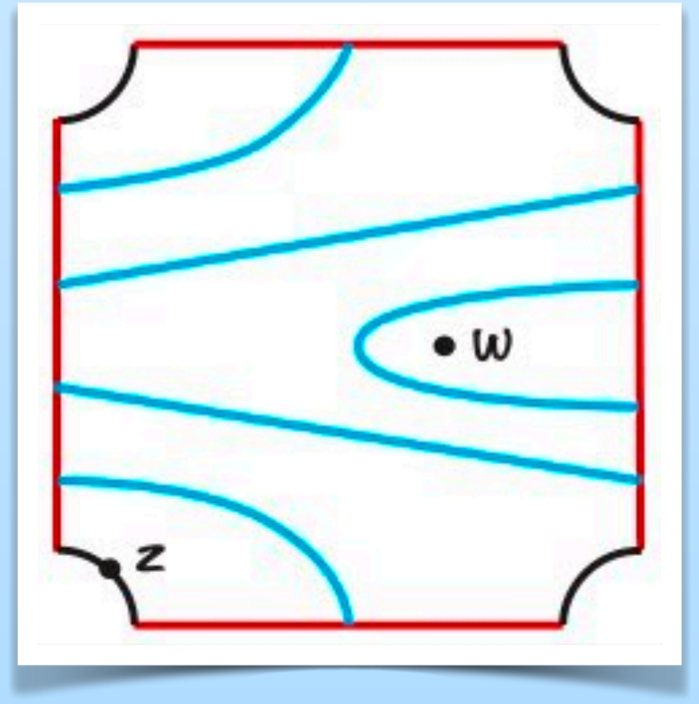
Theorem by example: $(T_{2,3})_{3,1}$

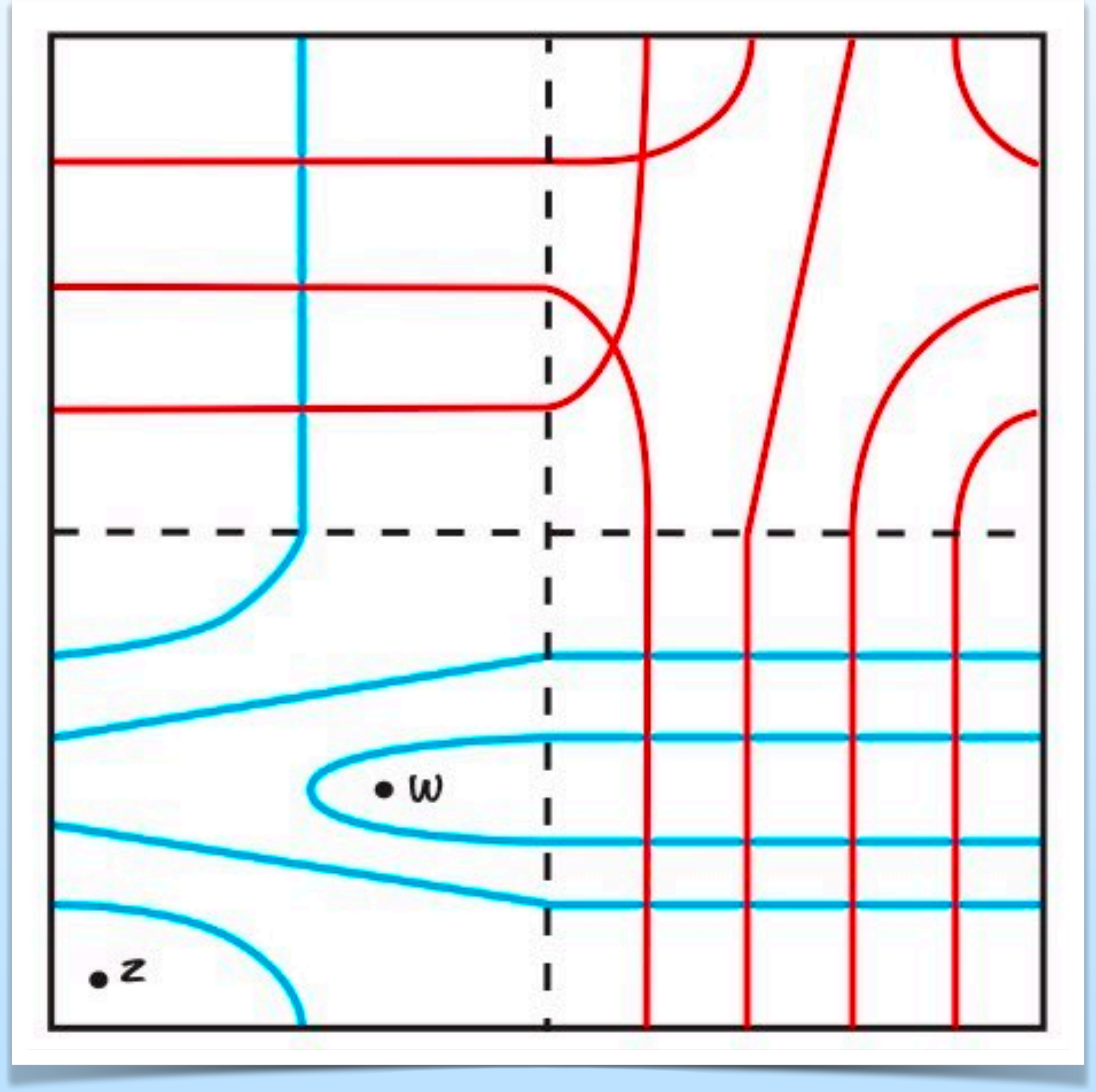
a genus one doubly pointed bordered Heegaard diagram for $(D^2 \times S^1, P)$



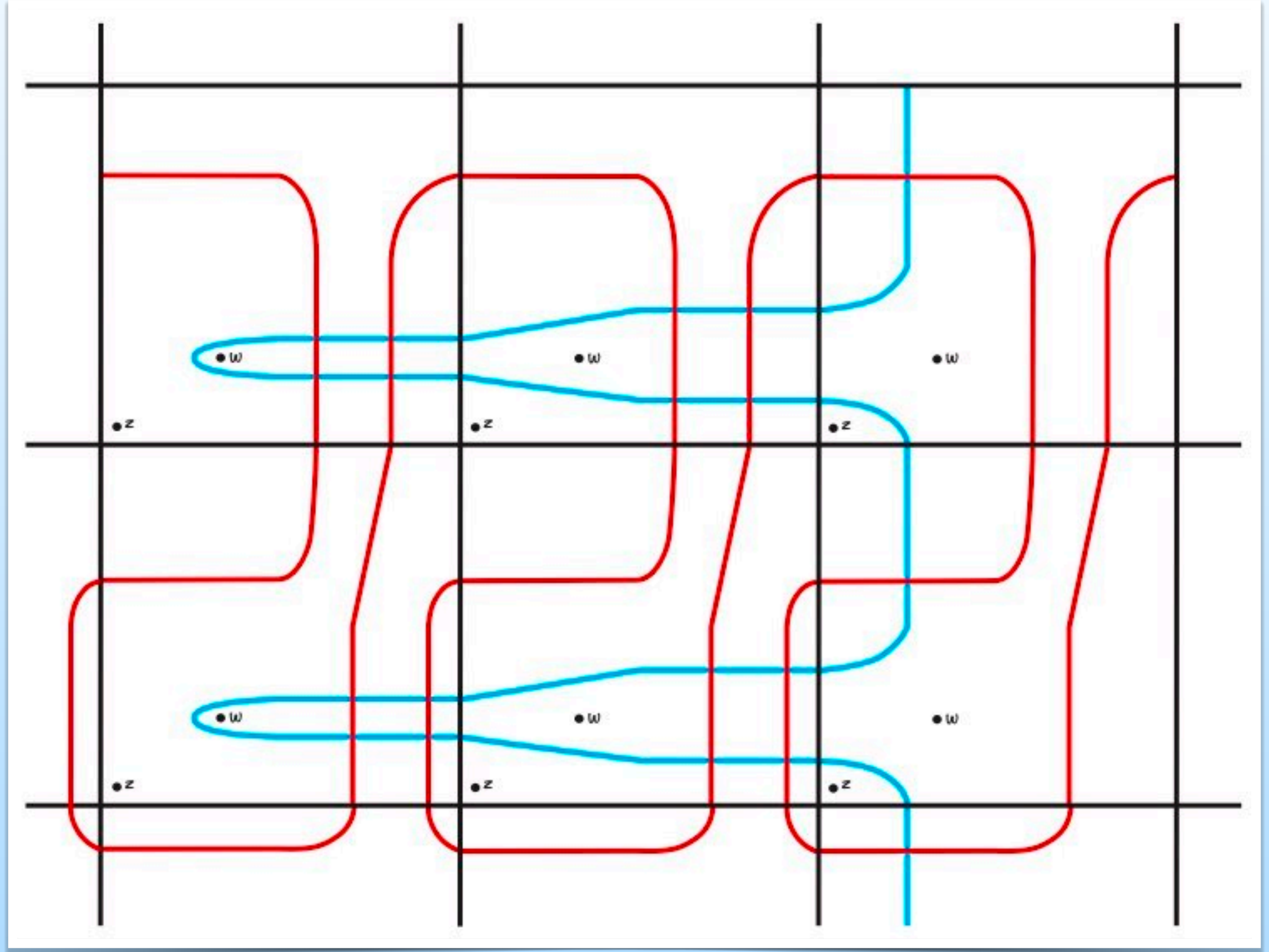
immersed curves for K in the punctured torus

T^2

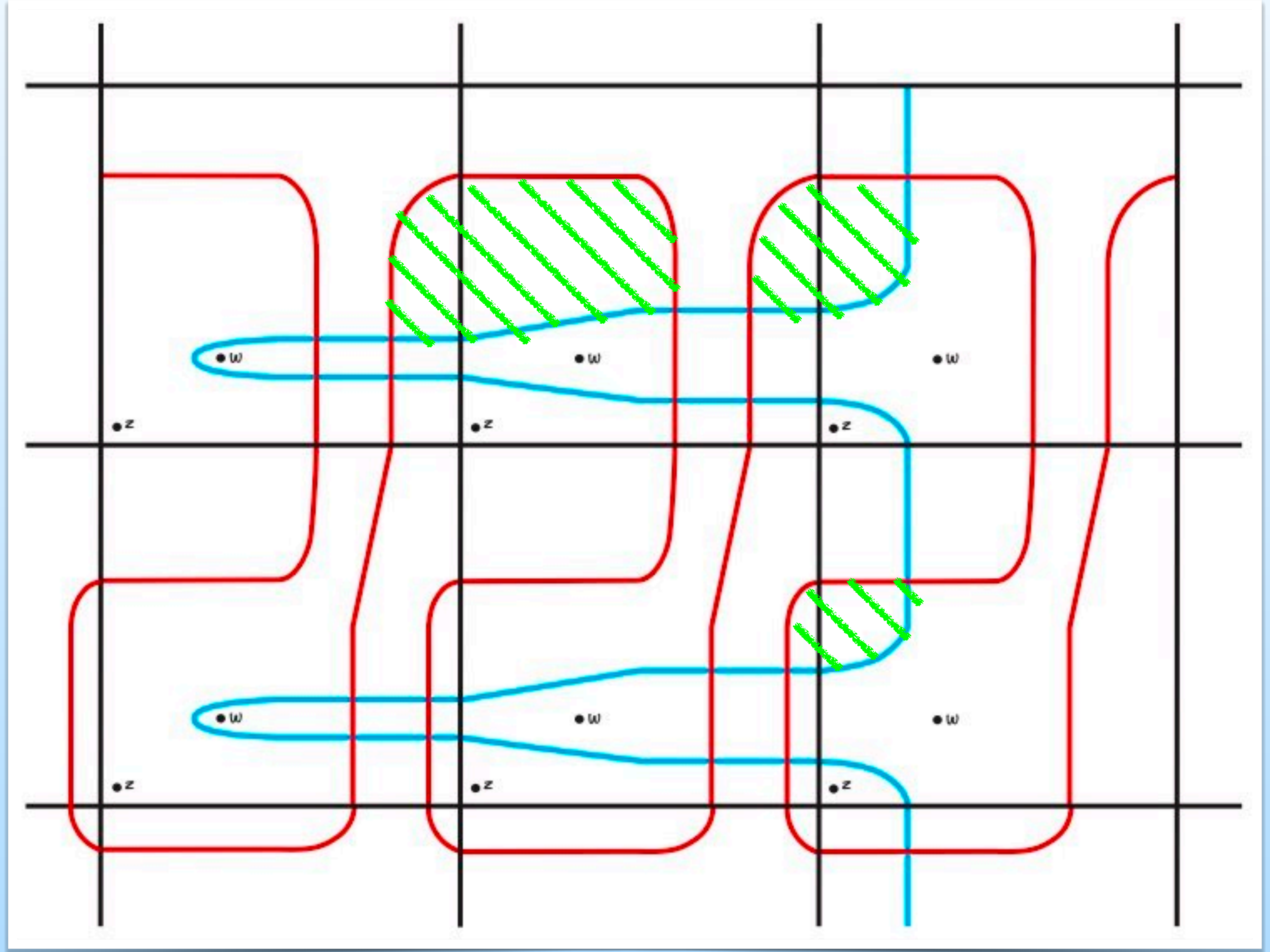




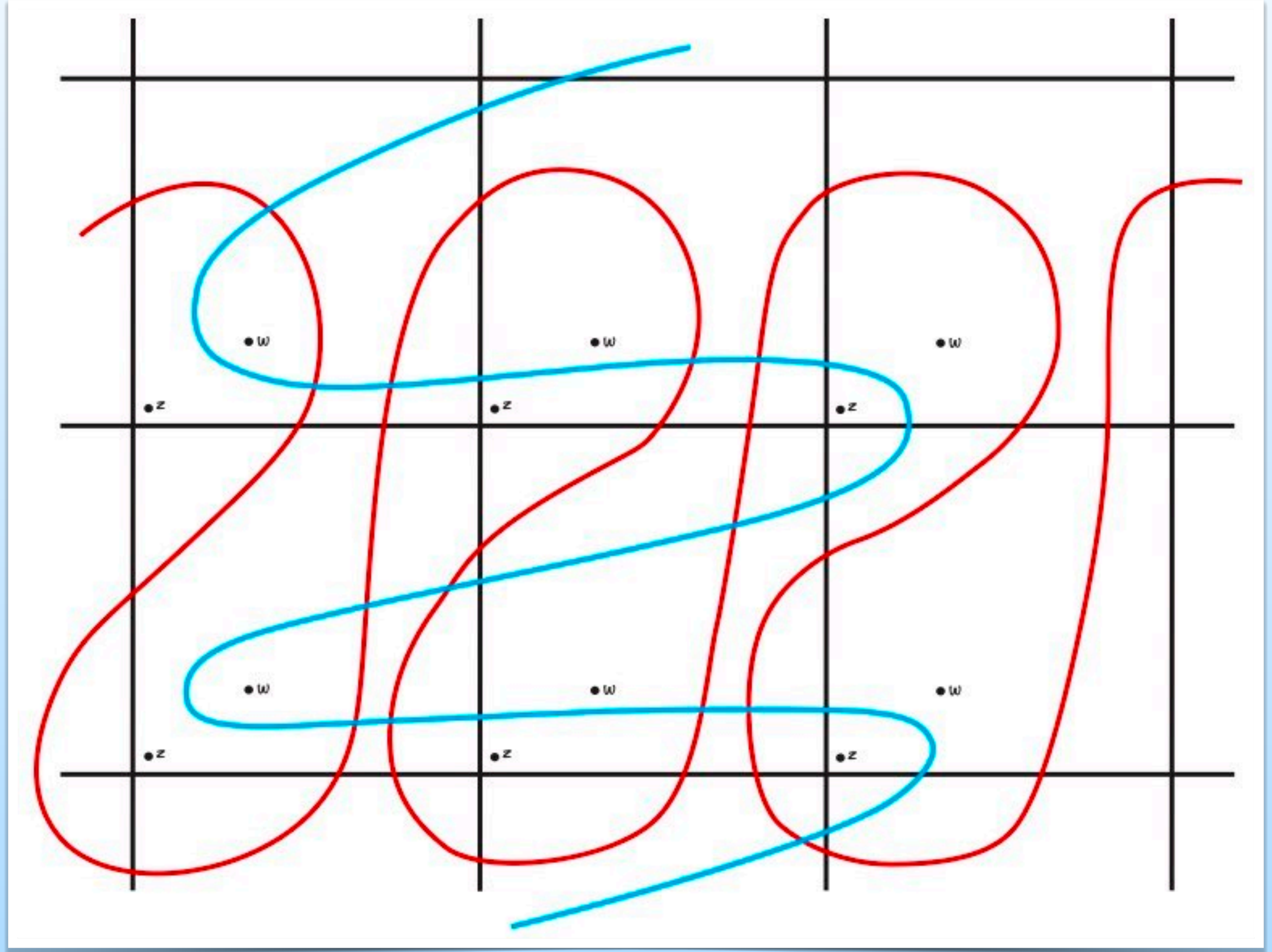
$\mathbb{R}^2$



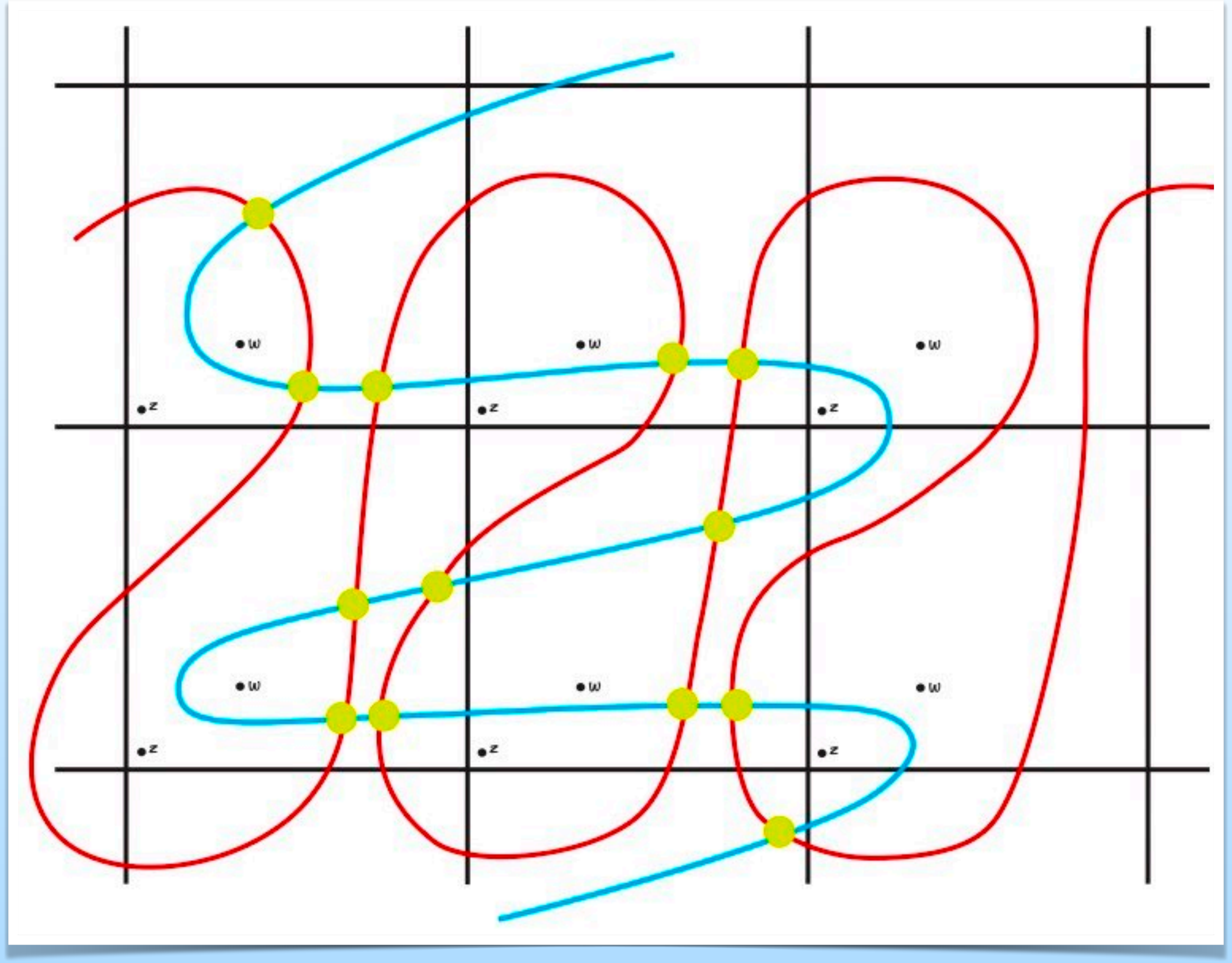
Bigons



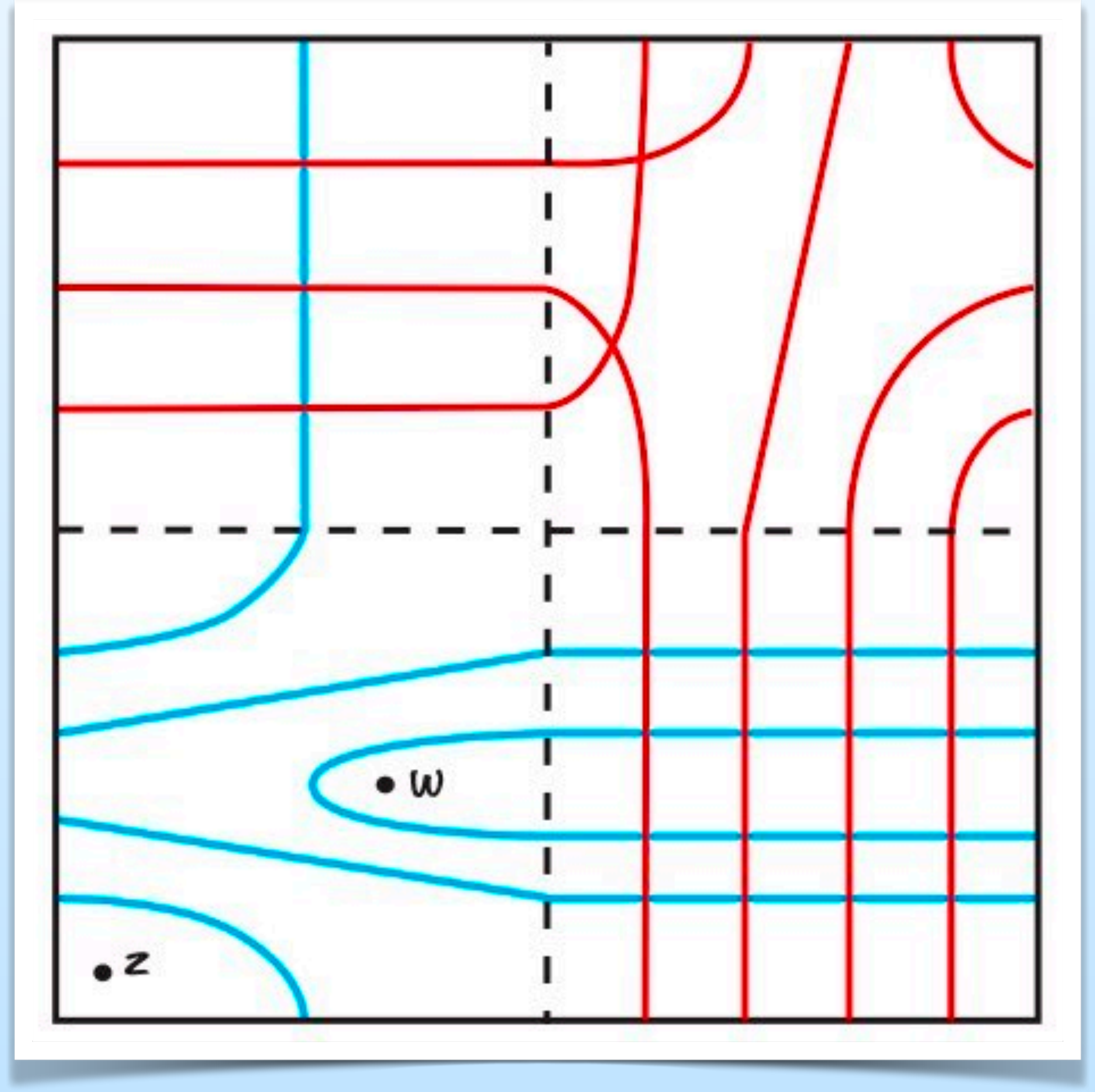
Minimum Intersections



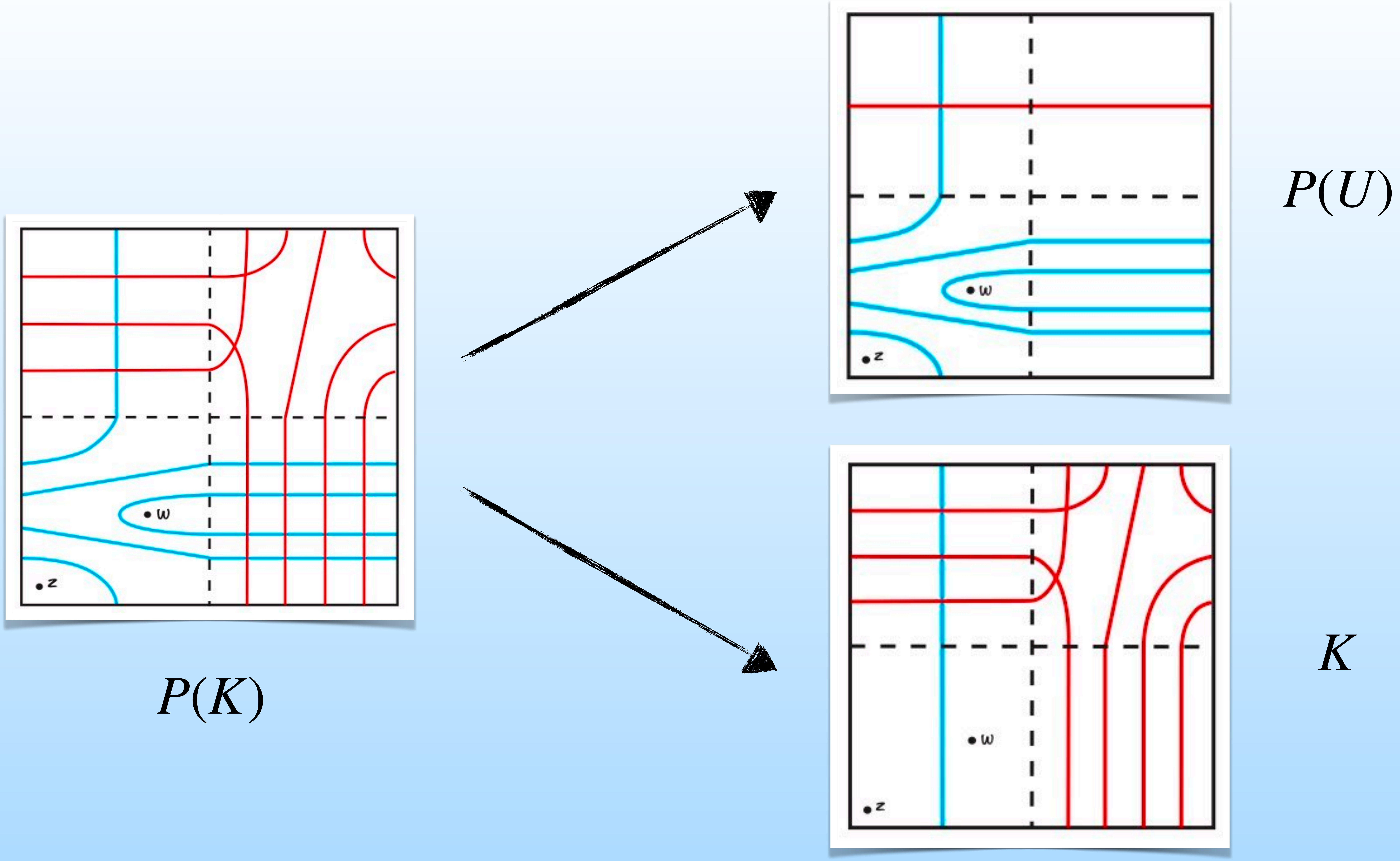
Theorem (W. Chen)



rank $\widehat{HFK}((T_{2,3})_{3,1}) = 13$



$P(K)$



w
z
P(K)
w
z
P(U)
w
z
K

Thank you!