Fraud-proof non-market allocation mechanisms

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The allocation problem

- **Problem**: A designer seeks to allocate $\rho \leq 1$ mass homogeneous objects to unit mass of heterogeneous agents without transfers
- **Examples:** vaccines, ICU beds, public housing, food stamps, organs, public school seats
- agent characteristics: $\theta = (i, s, k)$
- $i \in I$, I finite, publicly observable, unfalsifiable label (label can include age, gender, DOB, race); $\mu_i > 0$ mass of agents with label i
- $s \in [\underline{s}_i, \bar{s}_i]$ agent's natural score; **hard information** can be falsified at a cost
- dimensions k **soft information**; agents can misrepresent freely; include:
  - $v \geq 0$: private willingness to pay
  - parameters such as gaming ability affecting falsification cost

The allocation problem, continued

hybrid model with soft and hard info: $\theta = (i, s, k)$ can freely misrepresent soft dimensions and report k' but falsifying to t is costly; $C^\theta(s) = 0$ (costless not to falsify)

$$\text{Agents payoff} = \underbrace{\mathbb{P}(k',\ \text{submitting score } t|\theta)}_{\text{interim (exp.) prob. of obtaining an object}} \ v - \underbrace{C^\theta(t)}_{\text{cost of falsifying to score } t \text{ for } \theta=(i,s,k)}$$

$$\text{Designer's payoff} = \begin{cases} w \in \mathbb{R} \text{ from assigning an item to agent=agent's social value: unknown} \\ 0 \text{ from not assigning an item} \end{cases}$$

- (θ, w) IID draws across agents from a full support joint distribution (correlation possible across dimensions; indep. info across agents)
- if θ sufficient statistic for w agents know w
- conditional group social value: $w_i(s) = \mathbb{E}(w|i, s)$ is increasing in s $\rightarrow$ positive correlation between score and social value; assume is bounded and integrable
- conditional score distribution: $s \sim F_i(s)$ conditional on i with support $[\underline{s}_i, \overline{s}_i]$

Scores

Contexts

- **school choice; public housing**
  - natural score= priority based on true characteristics
  - falsified score= priority based on false characteristics: eg. fake address, false evidence of housing need
- **organ transplant**
  - natural score= priority based on true health status
  - falsified score= priority based on manipulated status based on escalated treatments
- **admission to selective schools/ colleges**
  - natural score= based on true talents
  - falsified score= based on cheating, fake evidence of athletic ability (cf. Varsity blues)

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Broadly: in various contexts there exists an exogenous technology measuring and aggregating agent's characteristics into a one-dimensional *score* / *metric* / *priority*. An agent's *natural score*–the one she obtains when she does not interfere with the measuring technology–may differ from her *measured score* when she engages in *gaming* / *manipulations* / *falsification* / *cheating*.

Designer's objective

Allocate goods to maximize social value via falsification-proof non-market mechanisms

- Nonmarket $=$ monetary transfers are not available (human organs, public school seats)
- Falsification-proofness analogous to truth-telling; strategy-, manipulation-proofness
- however, with costly falsification, and more than two scores mechanisms maximizing social value may have to induce fraud– Perez-Richet and Skreta (2022)
- $\rightarrow$ imposing falsification-proofness is costly for the principal.

Why impose falsification proofness?

1. Mechanisms (especially nonmarket) inducing fraud come under scrutiny; difficult to defend (political / stability argument).

   Authorities in Boston and Chicago abandoned the "Boston" school assignment mechanism citing concerns about its vulnerability to manipulation.

   "The Boston episode challenges a paradigm in traditional mechanism design that treats incentive compatibility only as a constraint and not as a direct design objective, at least for the specific context of school choice."

   Pathak and Sonmez AER 2013

2. Fraud creates negative social externalities: dishonest and corrupt behavior erodes public trust

   Transplant providers in Germany were convicted for manipulating the liver allocation system by significantly exaggerating their patients' illness severity. Following this scandal, public confidence in the system eroded resulting to a 20%-40% decline in the number of organ donations and in the overall organ transplants performed (Bolton, 2018). (See also Galbiati and Zanella (2012); Ajzenman (2018); Alm et al. (2017); Rincke and Traxler (2011)).

Why impose falsification proofness?

3. Fraud is costly to the agents, and may generate lack of fairness (because of heterogeneous gaming abilities).

   "A strategy-proof algorithm "levels the playing field" by diminishing the harm done to parents who do not strategize or do not strategize well."

   BPS Strategic Planning Team

   Bjerre-Nielsen et. al. (2023) find that applicants from more affluent households drive address manipulations affecting non-manipulating applicants: more than 25% of honest agents would have been offered a place in their first choice absent of address manipulation and worse peers. Manipulations are also detrimental–and, in fact, seem to cost lives of non-manipulating agents–in human transplant settings (Bolton, 2018).

4. Falsification proofness protects the integrity of scores

   Goodhart's law predicts "when a measure becomes a target, it ceases to be a good measure."

Score-based allocation mechanisms

Falsification-proof non-market mechanisms

1. cannot condition on reports about "soft" dimensions of type; all agents with same i, s behave the same way regardless of k
2. need not condition on a continuum of scores; because there is a continuum of agents interim allocation probabilities satisfy suitable Border conditions

A non-market mechanism is therefore a score-based allocation rule $\alpha = (\alpha_i)_{i \in I}$ where

$$\alpha_i : [\underline{s}_i, \bar{s}_i] \to [0, 1]$$

A score-based allocation mechanism is falsification-proof iff

$$\alpha_i(s)v \geq \alpha_i(t)v - C^{(i,s,k)}(t) \quad \forall i, s, t, k$$

$\rightarrow$ constraint hardest to satisfy for high v-low cost agents; these agents shape the mechanism
lower bound on cost/value of manipulating to t for agents with natural score s in group i:

$$\frac{1}{\gamma_i} c_i(t|s) \equiv \inf_{k \in K_{i,s}} \frac{1}{v} C^{(i,s,k)}(t), \text{for every } t, s \text{ in group } i$$

Properties of lower bound of falsification cost

We assume bound is tight and:

- $c_i(t|s)$ is measurable and increasing for *upward* falsifications: if $t \geq s$, then $c_i(t|s)$ is (locally) strictly increasing in t and $-s$
- regularity (REG): $c(t|s)$ is regular if it is continuously differentiable in t on $[s, \bar{s}]$, and in s on $[\underline{s}, t]$, and there exists $\Lambda > 0$ such that, for every s, t, $c(t|s) \leq \Lambda|t - s|$.

Planner's problem

$$\max_{\alpha} \sum_i \mu_i \int \alpha_i(s) w_i(s) dF_i(s)$$

$$\text{s.t.} \sum_i \mu_i \int \alpha_i(s) dF_i(s) \leq \boldsymbol{\rho} \text{ resource constraint} \quad \text{(RC)}$$

$$\mu_i \int \alpha_i(s) dF_i(s) \geq \phi_i \boldsymbol{\rho} \quad \forall i \text{ quota constraints} \quad \text{(QC)}$$

$$0 \leq \alpha_i(s) \leq 1 \quad \forall i, s \text{ probability constraints} \quad \text{(PROB)}$$

$$\alpha_i(s) \geq \alpha_i(t) - \frac{1}{\gamma_i} c_i(t|s) \quad \forall i, s, t \text{ falsification-proofness constraints} \quad \text{(FPIC)}$$

Within and across decompositions

within problem: how to optimally allocate mass ρ_i group i:

$$W_i(\rho_i) = \max_{\alpha_i} \int_{S_i} \alpha_i(s) w_i(s) dF_i(s) \tag{P}$$

$$\text{s.t. (FPIC),(PROB)}$$

$$\mu_i \int_{S_i} \alpha_i(s) dF_i(s) = \rho_i, \tag{RC}$$

across problem: optimally determine $(\rho_i)_{i \in I}$ subject to feasibility $R = \left\{\boldsymbol{\rho} : \sum_i \rho_i \leq \bar{\rho},\ \rho_i \geq \phi_i \bar{\rho}\ (\forall i)\right\}$

$$\overline{W}(\boldsymbol{F}, \gamma) = \max_{\boldsymbol{\rho} \in R} \sum_i \mu_i W_i(\rho_i). \tag{$\bar{\text{P}}$}$$

Within problem: determining eligibility threshold

Lagrangian:

$$\int \alpha_i(s)[w_i(s) - \hat{w}_i]dF_i(s) + K(\hat{w}_i)$$

- Lagrange multiplier defines a planner outside options $\hat{w}_i$; adjusts to satisfy the resource constraint
- $\hat{w}_i$ determines endogenous prioritization: group i:

$$\overline{w}_i \equiv \int w_i(s)dF_i(s) \begin{cases} > \hat{w}_i \text{ has high priority} \\ < \hat{w}_i \text{ it has low priority} \\ = \hat{w}_i \text{ it has neutral priority} \end{cases}$$

The auxiliary within group problem

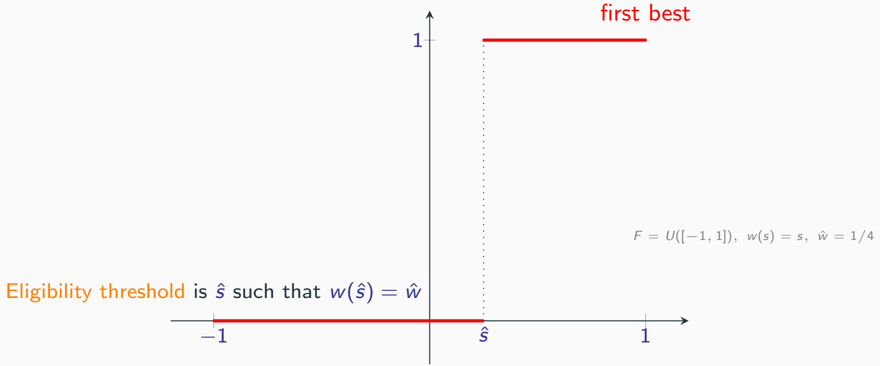
$$\max_{\alpha} \int \alpha(s)\{w(s) - \hat{w}\}dF(s)$$

$$\text{s.t.} \quad \alpha(t) - \alpha(s) \leq \frac{1}{\gamma} c(t|s) \quad \forall s, t \qquad \text{(FPIC)}$$

$$0 \leq \alpha(s) \leq 1 \quad \forall s \qquad \text{(PROB)}$$

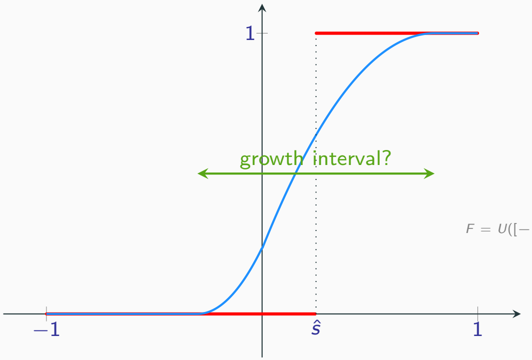
First-best

$$\max_{\alpha} \int \alpha(s)\{w(s) - \hat{w}\} dF(s) \text{ subject to } 0 \leq \alpha(s) \leq 1 \quad \forall s$$



Note: With transfers this is typical shape of optimal allocations. Sudden increase in payment makes this allocation incentive compatible. Not possible without transfers!

Optimal FPIC rule?



$F = U([-1, 1]),\ w(s) = s,\ \hat{w} = 1/4$

Optimal allocation WLOG monotonic and Lipschitz continuous: Lipschitz continuity is implied by (FPIC) and (REG); monotonicity: take a candidate α for $s < \hat{s}$ use highest increasing function everywhere below α while for $s > \hat{s}$ use lowest increasing function everywhere above the candidate α

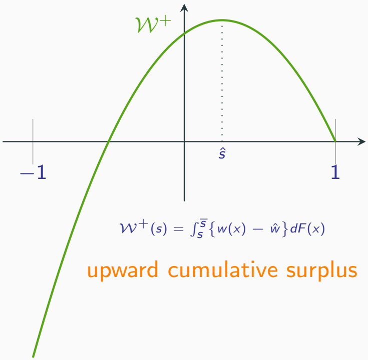
$$\alpha(s) = \underline{\alpha} + \int_{\underline{s}}^{s} \alpha'(z)dz = \overline{\alpha} - \int_{s}^{\overline{s}} \alpha'(z)dz$$

Cumulative surplus and growth intervals

Low priority group: $\overline{w} < \hat{w}$

$$\max_{\alpha \text{ s.t PROB, FPIC}} \quad \underline{\alpha}(\overline{w} - \hat{w}) + \int \alpha'(s)\mathcal{W}^{+}(s)ds$$

$F{=}U([-1,1]),\ w(s){=}s,\ \hat{w}{=}1/4$



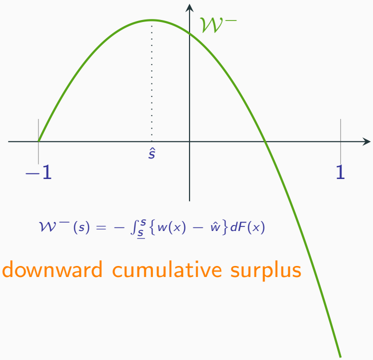
$\mathcal{W}^{+}(s) = \int_{s}^{\overline{s}} \{w(x) - \hat{w}\} dF(x)$

upward cumulative surplus

High priority group: $\overline{w} \geq \hat{w}$

$$\max_{\alpha \text{ s.t PROB, FPIC}} \quad \overline{\alpha}(\overline{w} - \hat{w}) + \int \alpha'(s)\mathcal{W}^{-}(s)ds$$

$F{=}U([-1,1]),\ w(s){=}s,\ \hat{w}{=}-1/4$



$\mathcal{W}^{-}(s) = -\int_{\underline{s}}^{s} \{w(x) - \hat{w}\} dF(x)$

downward cumulative surplus

$\mathcal{W}^{+}(s)$= marginal gain of uniformly increasing allocation prob of all scores above s; $\mathcal{W}^{-}(s)$=marginal gain of uniformly decreasing allocation prob of all scores below s

common (all priorities) differential program

$$
\begin{aligned}
\max_{\alpha'} \quad & \int \alpha'(s)\mathcal{W}(s)ds \\
\text{s.t.} \quad & 0 \le \alpha'(s), \quad \forall s && \text{(MON)} \\
& \int_s^t \alpha'(z)dz \le \frac{1}{\gamma}c(t|s), \quad \forall s < t && \text{(FPIC)} \\
& \int \alpha'(z)dz \le 1 && \text{(PROB)}
\end{aligned}
$$

where

$$
\mathcal{W}(s) = \mathcal{W}^+(s)\mathbb{1}_{\overline{w}<\hat{w}} + \mathcal{W}^-(s)\mathbb{1}_{\overline{w}\ge\hat{w}}
$$

common (all priorities) differential program

$$\max_{\alpha'} \quad \int \alpha'(s)\mathcal{W}(s)ds$$

$$\text{s.t.} \quad 0 \leq \alpha'(s), \quad \forall s \qquad \text{(MON)}$$

$$\int_s^t \alpha'(z)dz \leq \frac{1}{\gamma}c(t|s), \quad \forall s < t \qquad \text{(FPIC)}$$

$$[\nu] \quad \int \alpha'(z)dz \leq 1 \qquad \text{(PROB)}$$

where

$$\mathcal{W}(s) = \mathcal{W}^+(s)\mathbb{1}_{\overline{w}<\hat{w}} + \mathcal{W}^-(s)\mathbb{1}_{\overline{w}\geq\hat{w}}$$

common (all priorities) differential program

$$\max_{\alpha'} \int \alpha'(s)\{\mathcal{W}(s) - \nu\} ds + K(\nu)$$

$$\text{s.t.} \quad 0 \leq \alpha'(s), \quad \forall s \tag{MON}$$

$$\int_s^t \alpha'(z) dz \leq \frac{1}{\gamma} c(t|s), \quad \forall s < t \tag{FPIC}$$

$\Rightarrow$ So $\alpha'(s) = 0$ on $\{s : \mathcal{W}(s) < \nu\}$. where

$$\mathcal{W}(s) = \mathcal{W}^+(s) \mathbb{1}_{\overline{w} < \hat{w}} + \mathcal{W}^-(s) \mathbb{1}_{\overline{w} \geq \hat{w}}$$

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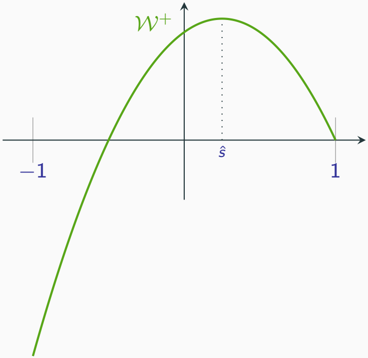

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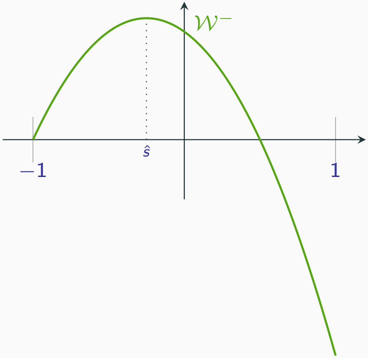

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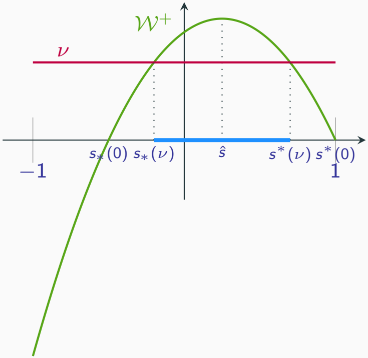

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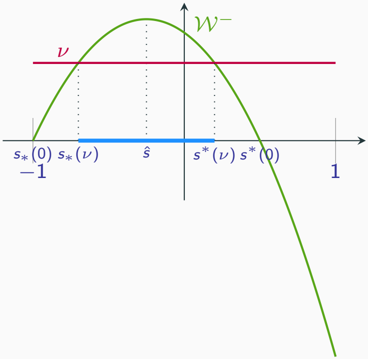

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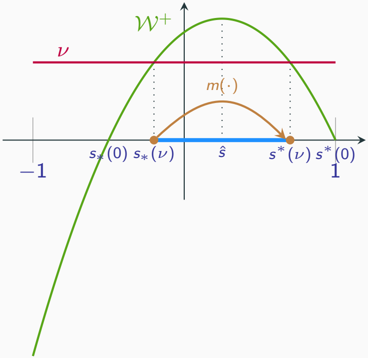

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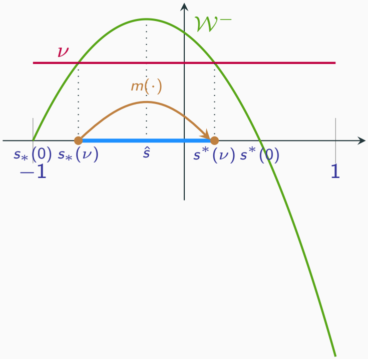

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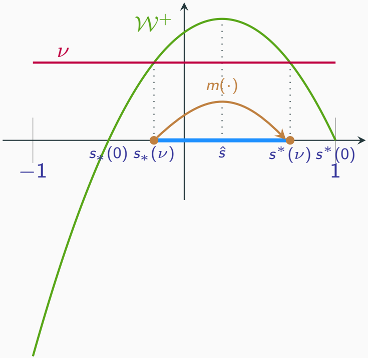

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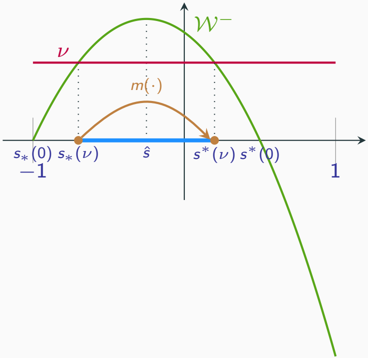

matching function $m : [s_*(0), \hat{s}] \to [\hat{s}, s^*(0)]$ s.t. $\mathcal{W}(s) = \mathcal{W}\big(m(s)\big) \implies \int_s^{m(s)} \{w(x) - \hat{w}\} dF(x) = 0$

Reduced; relaxed program

$$\max_{\alpha} \int_{s_*}^{s^*} \alpha'(s)\mathcal{W}(s)dF(s)$$

$$\text{s.t. } \alpha(t) - \alpha(s) \leq \frac{1}{\gamma}c(t|s) \quad \forall\ s_* \leq s < t \leq s^* \tag{FPIC}$$

where $s_* \leq \hat{s} \leq s^*$ and $\mathcal{W}(s_*) = \mathcal{W}(s^*) \geq 0$.

Procedure:

1. Solve the reduced program for any (s_*, s^*).
2. Select (s_*, s^*) such that:
   - $\alpha(s^*) - \alpha(s_*) = 1$ (PROB) binds, or
   - $[s_*, s^*] = [s_*(0), s^*(0)]$; $\nu = 0$ (PROB) does not bind.

Fraud technologies

Upward Increasing Differences

$$\forall s < s' \leq t < t', \quad c(t'|s) - c(t|s) \leq c(t'|s') - c(t|s') \tag{UID}$$

Example: Euclidean cost

$c(t|s) = \mathcal{C}(|t-s|)$, $\mathcal{C}$ concave: increasing returns to scale of fraud.

Upward Decreasing Differences

$$\forall s < s' \leq t < t', \quad c(t'|s) - c(t|s) \geq c(t'|s') - c(t|s') \tag{UDD}$$

Example: Euclidean cost

$c(t|s) = \mathcal{C}(|t-s|)$, $\mathcal{C}$ convex: decreasing returns to scale of fraud.

Fraud technologies

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FPIC bind for far apart scores $\rightarrow$ new method based on optimal transport.

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FPIC bind locally $\rightarrow$ first-order approach.

Optimal transport approach: UID

$$\max_{\alpha} \int_{s_*}^{\hat{s}} \alpha(s)\{w(s) - \hat{w}\} dF(s) + \int_{\hat{s}}^{s^*} \alpha(t)\{w(t) - \hat{w}\} dF(t)$$

$$\text{s.t.} \quad \alpha(t) - \alpha(s) \leq \frac{1}{\gamma} c(t|s), \quad \forall s_* \leq s < t \leq s^* \tag{FPIC}$$

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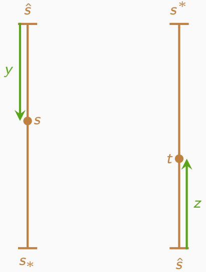

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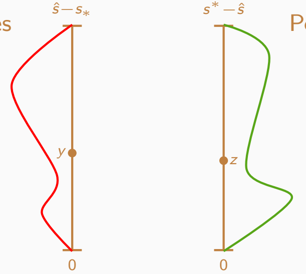
Negative social surplus scores

$\mathcal{Y}$

$P(y) = \frac{\mathcal{W}(\hat{s}) - \mathcal{W}(\hat{s}-y)}{\mathcal{W}(\hat{s}) - \mathcal{W}(s_*)}$

$p(y) \propto -\{w(\hat{s}-y) - \hat{w}\}f(\hat{s}-y)$

$\phi(y) = \alpha(\hat{s}-y)$



Positive social surplus scores

$\mathcal{Z}$

$Q(z) = \frac{\mathcal{W}(\hat{s}) - \mathcal{W}(\hat{s}+z)}{\mathcal{W}(\hat{s}) - \mathcal{W}(s^*)}$

$q(z) \propto \{w(\hat{s}+z) - \hat{w}\}f(\hat{s}+z)$

$\psi(z) = \alpha(\hat{s}+z)$

Optimal transport approach: UID

$$\max_{\phi,\psi} \int_{\mathcal{Z}} \psi(z)dQ(z) - \int_{\mathcal{Y}} \phi(y)dP(y)$$

$$\text{s.t. } \psi(z) - \phi(y) \leq \frac{1}{\gamma}c(\hat{s}+z|\hat{s}-y), \quad \forall y,z \qquad \text{(FPIC)}$$

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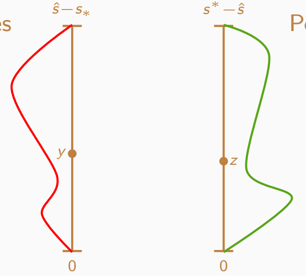

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$q(z)\propto\{w(\hat{s}+z)-\hat{w}\}f(\hat{s}+z)$

$\psi(z)=\alpha(\hat{s}+z)$

Optimal transport approach: UID

Dual of the Monge-Kantorovich optimal transport problem:

$$\min_{G\in\mathcal{M}(P,Q)} \frac{1}{\gamma}\int_{\mathcal{Y}\times\mathcal{Z}} \underbrace{c(\hat{s}+z|\hat{s}-y)}_{\text{submodular}} dG(y,z)$$

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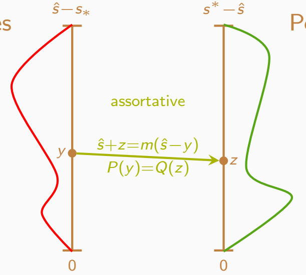
$p(y)\propto-\{w(\hat{s}-y)-\hat{w}\}f(\hat{s}-y)$

$\phi(y)=\alpha(\hat{s}-y)$



Positive social surplus scores

$\mathcal{Z}$

$Q(z)=\frac{\mathcal{W}(\hat{s})-\mathcal{W}(\hat{s}+z)}{\mathcal{W}(\hat{s})-\mathcal{W}(s^*)}$

$q(z)\propto\{w(\hat{s}+z)-\hat{w}\}f(\hat{s}+z)$

$\psi(z)=\alpha(\hat{s}+z)$

Solution of auxiliary problem for UID: α^*_{uid}

Low priority group: $\overline{w} < \hat{w}$

$$\alpha^*_{uid}(s,\hat{w}) = \begin{cases} 0 & \text{if } s < s_* \\ -\frac{1}{\gamma}\int_{s_*}^{s} c_s\left(m(x)|x\right)dx & \text{if } s \in [s_*, \hat{s}] \\ \frac{1}{\gamma}c(s^*|s_*) - \frac{1}{\gamma}\int_{s}^{s^*} c_t\left(x|m^{-1}(x)\right)dx & \text{if } s \in [\hat{s}, s^*] \\ 1 & \text{if } s > s^* \end{cases}$$

High priority group: $\overline{w} > \hat{w}$

$$\alpha^*_{uid}(s,\hat{w}) = \begin{cases} 0 & \text{if } s < s_* \\ 1 - \frac{1}{\gamma}c(s^*|s_*) - \frac{1}{\gamma}\int_{s_*}^{s} c_s\left(m(x)|x\right)dx & \text{if } s \in [s_*, \hat{s}] \\ 1 - \frac{1}{\gamma}\int_{s}^{s^*} c_t\left(x|m^{-1}(x)\right)dx & \text{if } s \in [\hat{s}, s^*] \\ 1 & \text{if } s > s^* \end{cases}$$

where:

$$s_* = \min\left\{s \in [s_*(0), \hat{s}] \, : \, \frac{1}{\gamma}c(m(s_*)|s_*) \leq 1\right\} \text{ and } s^* = m(s_*)$$

Neutral priority group: $\overline{w} = \hat{w}$ in this case:

- $\mathcal{W}^-(s) = \mathcal{W}^+(s)$
- $[s_*(0), s^*(0)] = [\underline{s}, \overline{s}]$
- prob. constraint binds iff $\frac{1}{\gamma}c(\overline{s}|\underline{s}) \geq 1 \iff \gamma \leq c(\overline{s}|\underline{s}) \equiv \bar{\gamma}_{uid}$
- any rationing of probability slack $1 - \frac{1}{\gamma}c(s^*|s_*)$ optimal (unique solution at all other cases)

Optimal within group allocation

Theorem For any $0 \leq \rho \leq \mu$, there exists a unique outside option value $\hat{w}(\rho)$ and, if $\hat{w}(\rho) = \bar{w}$ and $\gamma > \bar{\gamma}$, a unique value $r(\rho)$, such that $\mu \int \alpha^*(s, \hat{w}, r) dF(s) = \rho$. Furthermore, $\hat{w}(\rho)$ is continuous, nonincreasing in ρ if $\hat{w}(\rho) \neq \bar{w}$ or $\gamma \leq \bar{\gamma}$, and constant at $\bar{w}$ otherwise. The function $r(\rho)$ is continuous and decreasing. The allocation rule $\alpha^*(s, \hat{w}(\rho), r(\rho))$ is then the unique solution to the within problem (P). The value function of (P), $W(\rho)$ is strictly concave at ρ if $\hat{w}(\rho) \neq \bar{w}$ or $\gamma \leq \bar{\gamma}$.

Remarks

1. at a solution $\hat{w}$ and α jointly adjust as a function of the mass of objects ρ the planner seeks to allocate to i
2. the higher the ρ the lower the $\hat{w}$

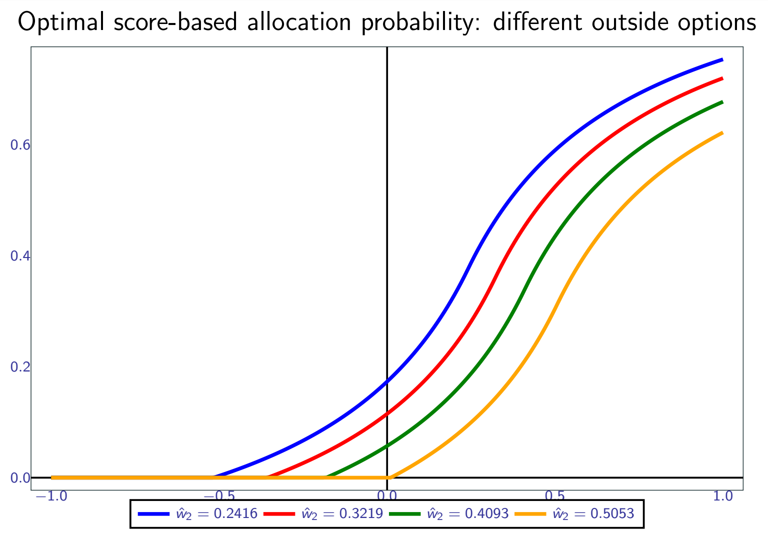


Figure 1: Cost $\gamma c(t|s) = \gamma|t - s|/(1 + |t - s|)$ if $t \geq s$, and $F_i = U(-1, 1)$

Optimal across group allocation

Endogenize $\boldsymbol{\rho} = \{\rho_i\}_{i \in I}$

$$\overline{W}(\boldsymbol{F}, \gamma) = \max_{\boldsymbol{\rho} \in R} \sum_i \mu_i W_i(\rho_i), \tag{$\overline{\text{P}}$}$$

Theorem The across problem ($\overline{\text{P}}$) admits a solution $\boldsymbol{\rho}$. Furthermore, $\boldsymbol{\rho}$ solves the across problem if and only if there exist a scalar $\lambda_R \geq 0$ and, for each i, a scalar $\lambda_i \geq 0$, an outside option value $\hat{w}_i(\rho_i)$, and a gap share $r_i(\rho_i)$ such that:

(i) $\lambda_i(\phi_i \bar{\rho} - \rho_i) = 0$ for all i

(ii) $\lambda_R(\sum_i \rho_i - \bar{\rho}) = 0$

(iii) $\hat{w}_i(\rho_i) = \lambda_R - \lambda_i$.

(iv) $\mu_i \int a_i^*(s_i, \hat{w}_i(\rho_i), r_i(\rho_i)) dF_i(s_i) = \rho_i$

The solution $\boldsymbol{\rho}$ is unique if, for each i, $\hat{w}(\rho_i) \neq \bar{w}_i$ or $\gamma_i \leq \bar{\gamma}_i$.

Properties & comparative statics

Properties of optimal assignment rules

- optimal mechanism involves inefficiencies: rations eligible agents; assigns objects to ineligible ones
- random assignment not optimal; ration agents with score-dependent probability
- scores below $s_*(0)$ never assigned objects
- (UDD): Prior distribution affects only growth interval: the matching function determines $m(s_*(\nu))$
- (UID): Prior distribution affects the whole test curve through the matching function.
- a first-order stochastic dominance shift in F
  - benefits the principal
  - not necessarily the agent: it increases the allocation probability for all agents iff it decreases the matching function.
    - Sufficient condition: transform a mass of negative surplus scores into positive surplus scores

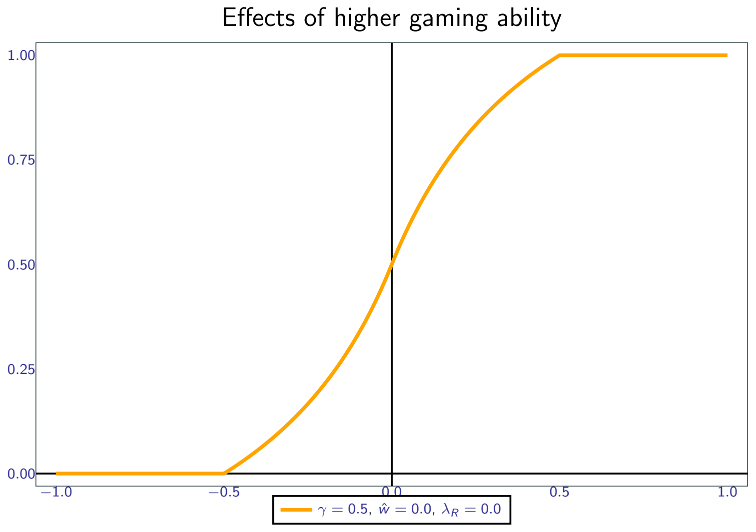
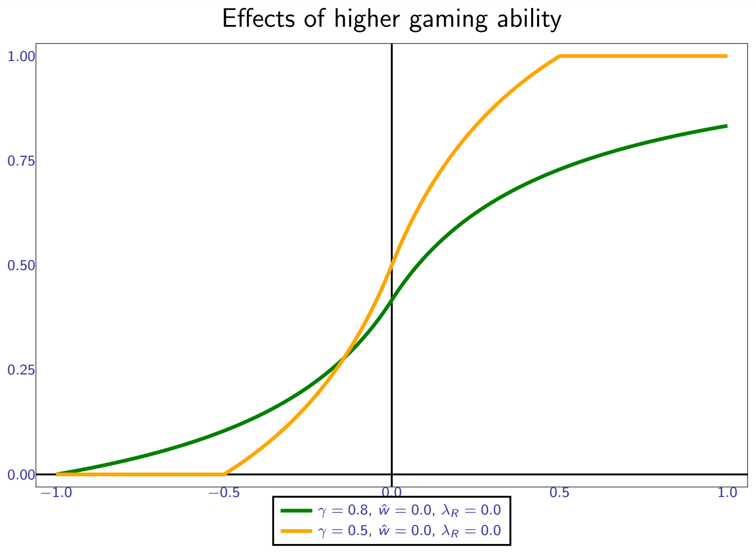


Figure 2: Cost $\frac{1}{\gamma}c(t|s) = \frac{1}{\gamma}\frac{|t-s|}{(1+|t-s|)}$ if $t \geq s$, and $F_i = U(-1, 1)$, $\rho = 0.5$



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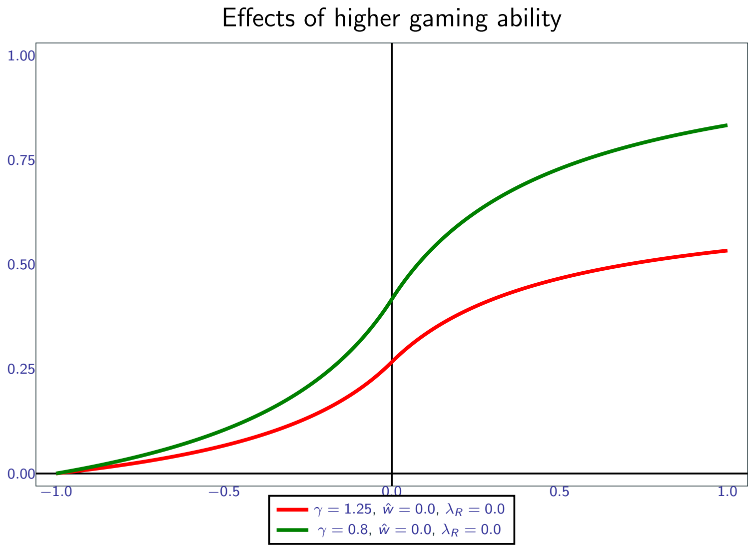

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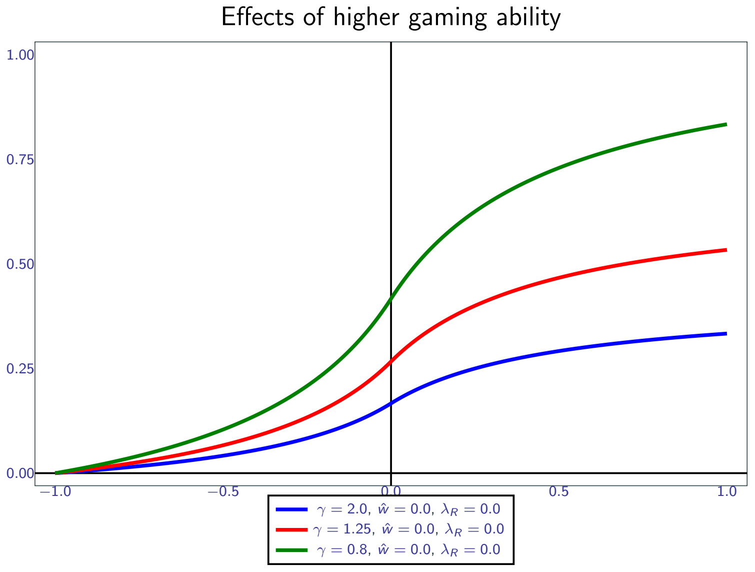


Figure 2: Cost $\frac{1}{\gamma}c(t|s) = \frac{1}{\gamma}\frac{|t-s|}{(1+|t-s|)}$ if $t \geq s$, and $F_i = U(-1,1)$, $\rho = 0.5$

Effects of gaming ability γ and falsification cost function

- planner's payoff is decreasing in γ
- agents (own-group) effects of γ: a threshold $\tilde{\gamma}$ exists such that a small increase in γ leads to:
  - If $\gamma < \tilde{\gamma}$, there exists a type threshold $\tilde{s}$ such that optimal the allocation probability $\alpha^*(s)$ increases for $s \leq \tilde{s}$, and decreases for $s \geq \tilde{s}$
  - If $\gamma > \tilde{\gamma}$, and the group has **high** priority, then $\alpha^*(s)$ increases for all scores
  - If $\gamma > \tilde{\gamma}$, and the group has **low** priority, then $\alpha^*(s)$ decreases for all scores
- Euclidean costs ($c(t|s) = \mathcal{C}(|t-s|)$) and returns to scale
  - If $\mathcal{C}$ is convex (UDD), the optimal allocation rule is linear with slope $\mathcal{C}'(0)$
  - If $\mathcal{C}$ is concave (UID), the optimal allocation rule is convex on $[s_*, \hat{s}]$ and concave on $[\hat{s}, s^*]$
  - Normalizing cost functions such that $\frac{1}{\gamma}\mathcal{C}(L) = 1$ for all $\mathcal{C}$. Then higher returns to scale (more concave $\mathcal{C}$) has the same effects as lower gaming ability γ (good for high scores bad for low scores)

Gaming ability across group effects

Conditioning on observables

Groups 1 gaming ability effects: scarce goods, group 2 20%

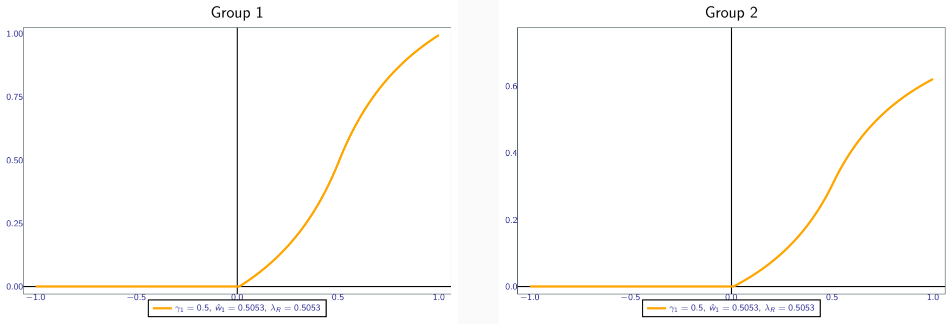
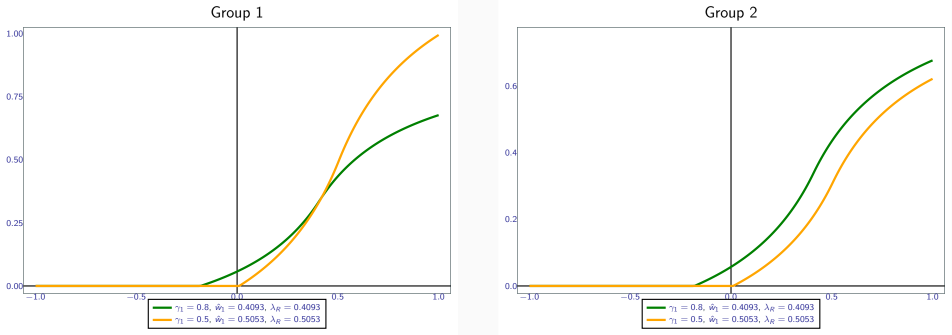


Figure 3: Cost $\frac{1}{\gamma}c(t|s) = \frac{1}{\gamma}\frac{|t-s|}{(1+|t-s|)}$, and $F_i = U(-1,1)$, $\phi_2 = 0.2$, $\gamma_2 = 0.8$, $\rho = 0.2$, $\mu_1 = \mu_2 = 0.5$

Group 1 higher gaming ability benefits group 2

Groups 1 gaming ability effects: scarce goods, group 2 20%



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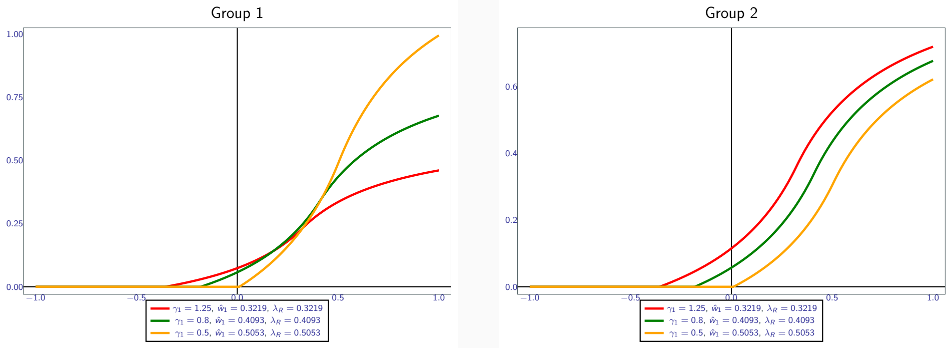

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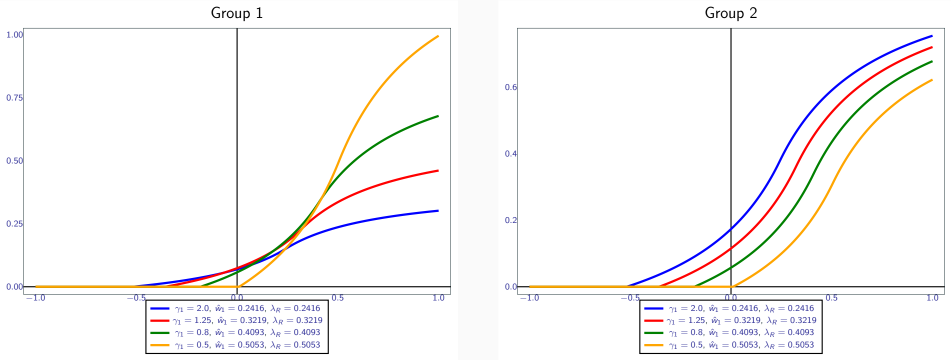
Group 1 higher gaming ability benefits group 2

Groups 1 gaming ability effects: scarce goods, group 2 20%



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Group 1 higher gaming ability benefits group 2

Groups 1 gaming ability effects: scarce goods, group 2 80% quota

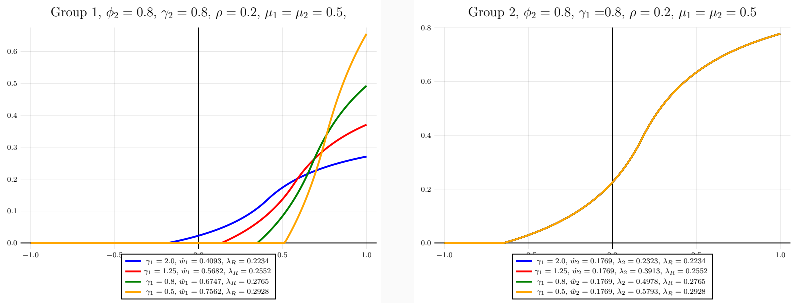

Figure 4: Cost $\gamma c(t|s) = \gamma|t-s|/(1+|t-s|)$ if $t \geq s$, and $F_i = U(-1,1)$

Group 1 lower gaming ability leaves unaffected group 2

Literature

Mechanism design; design of allocation mechanisms:
Myerson (1981), Akbarpour et al. (2020).

This paper: no transfers, costly state falsification

Mechanism design, allocation mechanisms without transfers:
Ben-Porath, Dekel and Lipman (2014), Mylovanov and Zapechelnyuk (2017), Kattwinkel (2020), Condorelli (2013).

This paper: exploit costly falsification

Literature

Mechanism design with costly misreporting / state falsification:

Deneckere and Severinov (2017), Kephart and Conitzer (2016), Lacker and Weinberg (1989), Landier and Plantin (2016), Severinov and Tam (2019)

This paper: no transfers

Lying Costs; communication under lying costs:

Abeler, Nosenzo and Raymond (2019), Gneezy, Kajackaite and Sobel (2020), Kartik (2009), Kartik, Ottaviani and Squintani (2009), Sobel (2020) This paper: mechanism design without transfers

Thanks