Mathematics, Computer Science and Economics of Market and Mechanism Design

Al Roth, Stanford University

MSRI/SLMath

September 11, 2023

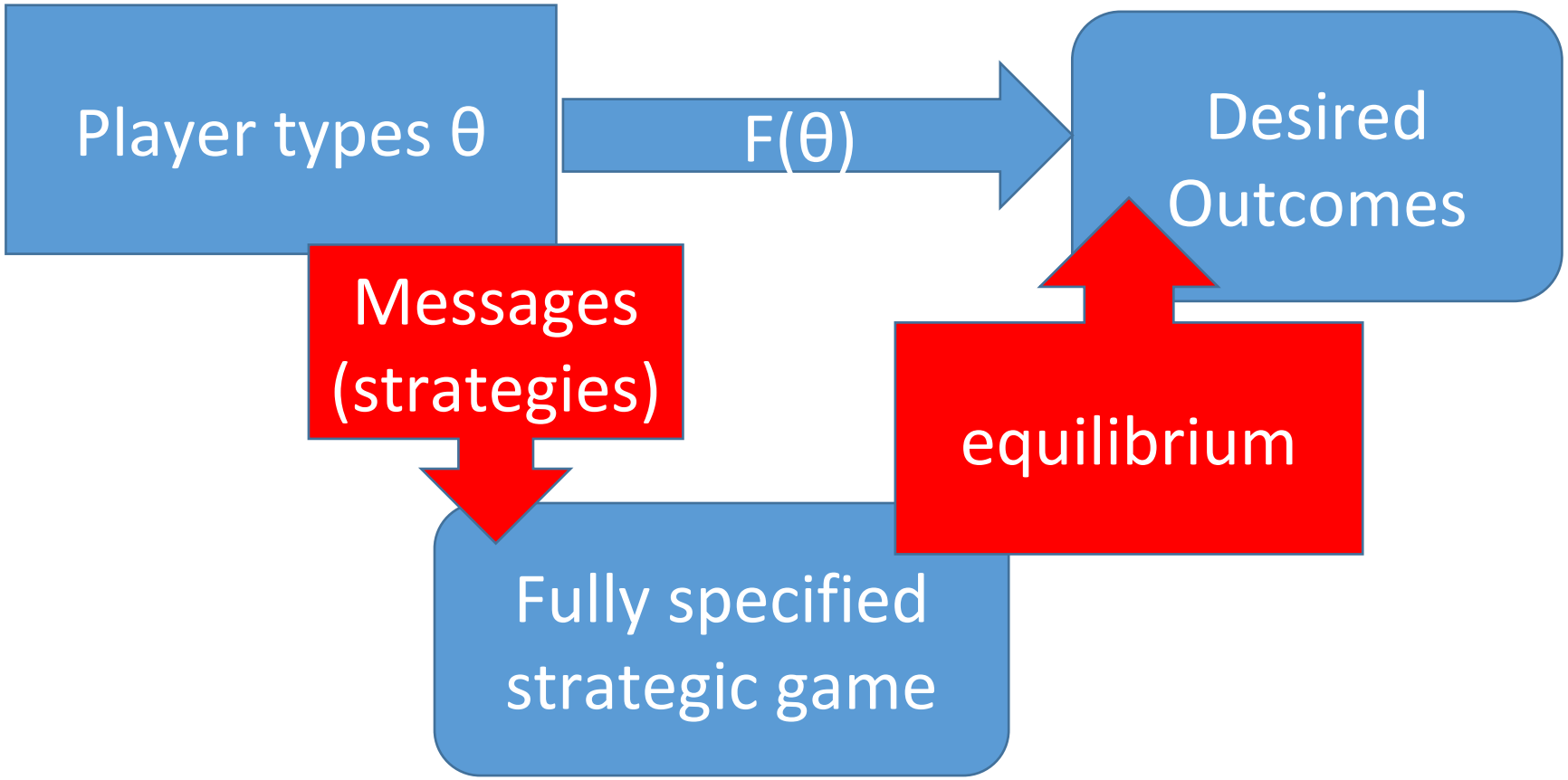
Welcome

- We're a gathering that includes economists, computer scientists, mathematicians and operations researchers. *Mechanism design* is in the intersection of those fields
- My talk today will be about mechanism design, and also about some parts of economics that typically fall outside of theoretical mechanism design but which inform a good deal of *practical market design*.

What do marketplaces do?

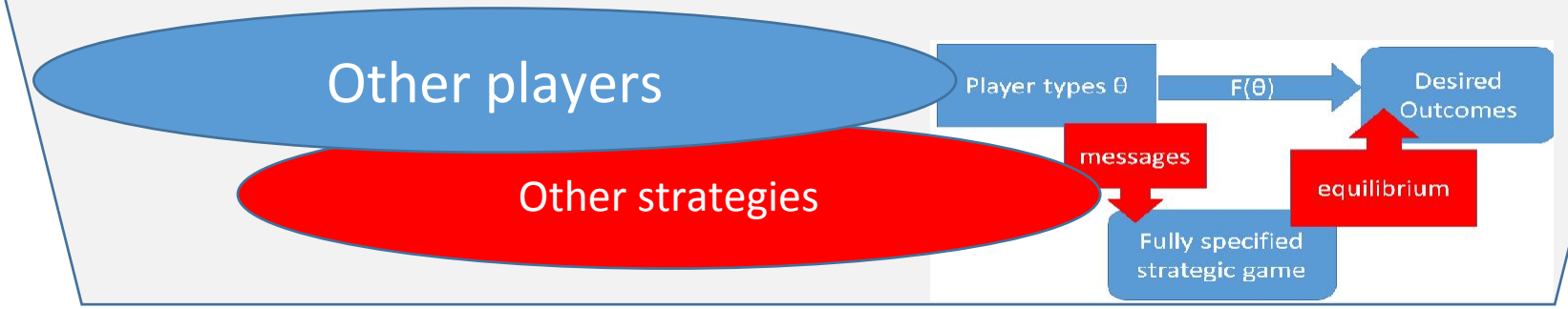
- They help markets become thick
- They manage the resulting congestion
- They create a safe environment to exchange information and conduct transactions

Mechanism Design



Market*place* Design

- Larger (Changing) Market Environment
  - Potential marketplace participants have large, *evolving* strategy sets outside of the marketplace “mechanism”



Cooperative and non-cooperative game theory have merged: coalitional and strategic models

- Prototypical theorems mixing cooperative and strategic models (these combine the work of multiple authors):
  - In [**some**] strategic model, when the direct mechanism using [**some**] algorithm is employed, it is a **dominant strategy** for [**some**] set of players to submit their true preferences, and the resulting outcome is [stable, in the core]

Matching markets

- Matching markets are markets in which you can’t just choose what you want (even if you can afford it), you also have to be chosen.

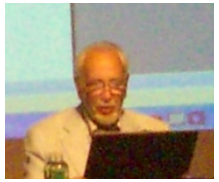
Venerable matching market design projects and their current renovations

- Market(s) for new doctors--residents and fellows--A century and a quarter, from 1900-2023
- Markets for new Ph.D. economists—50 years, from 1970's to today
- School choice, in NYC, Boston, Denver, DC, New Orleans…
- Kidney exchange (2000, 2004,--)
  - Continual adaptation
- And of course, AUCTIONS are iconic subjects of mechanism and practical market design

Background to redesign of the **medical clearinghouses**:

- 1900-1945 **UNRAVELLING OF APPOINTMENT DATES**
  - (like law clerks today, and some other markets)
- 1945-1950 CHAOTIC RECONTRACTING—**CONGESTION IN PROCESSING OFFERS**
- 1950-51 Development of a clearinghouse "mechanism"
- 1951-197x HIGH RATES OF ORDERLY PARTICIPATION

  (95%) in centralized clearinghouse
- 197x-198x DECLINING RATES OF PARTICIPATION (85%) particularly among the growing number of **MARRIED COUPLES**
- **1995-98** Market experienced a crisis of confidence with fears of substantial decline in orderly participation;
  - Design effort commissioned—to design and compare alternative matching algorithms capable of handling modern requirements: couples, specialty positions, etc.
  - **Roth-Peranson clearinghouse algorithm adopted, and employed**



Gale, David, and Lloyd S. Shapley. "College admissions and the stability of marriage." *American Mathematical Monthly* 1962.

Many-to-one matching: The college admissions model

PLAYERS: Firms = $\{f_1,\ldots, f_n\}$ Workers= $\{w_1,\ldots,w_p\}$

 # positions $q_1,\ldots,q_n$

Synonyms (sorry:-): F=Firms = C=Colleges = H=Hospitals

W=Workers = S=Students=D=Doctors

PREFERENCES *over individuals* (complete and transitive):

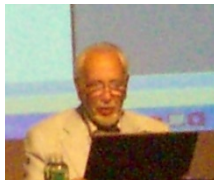
$P(f_i) = w_3, w_2, \ldots f_i \ldots$ $[w_3 >_{fi} w_2]$

$P(w_j) = f_2, f_4, \ldots w_j \ldots$

An OUTCOME of the game is a *MATCHING of firms and workers to one another.*: f is matched to the *set* of workers μ(f), each of whom is matched to f.

A matching is *stable* if there isn't any *blocking pair* consisting of a firm and worker not matched to each other but who would both prefer to be matched to each other than to (one of) their current matched partner(s)

Market	Stable	Still in use (halted unraveling)
• NRMP	yes	yes (new design in ’98)
• *Edinburgh ('69)*	*yes*	*yes*
• *Cardiff*	*yes*	*yes*
• *Birmingham*	*no*	*no*
• *Edinburgh ('67)*	*no*	*no*
• *Newcastle*	*no*	*no*
• Sheffield	no	no
• Cambridge	no	yes
• London Hospital	no	yes
• Medical Specialties	yes	yes (~30 markets, **1 failure**)
• Canadian Lawyers	yes	yes (Alberta, no BC, Ontario)
• Dental Residencies	yes	yes (5) (no 2)
• Osteopaths (< '94)	no	no
• Osteopaths (≥ '94)	yes	yes
• Pharmacists	yes	yes
• Reform rabbis	yes (first used in ‘97-98)	yes
• Clinical psych	yes (first used in ‘99)	yes
• Lab experiments	yes	yes.
(Kagel&Roth *QJE* 2000)	no	no



Basic **Deferred Acceptance** Algorithm

- Step 1: Each worker "applies" to his/her first choice. Each firm assigns its positions to its applicants one at a time in the firm's preference order until all positions are *tentatively* filled. Any remaining applicants are rejected.

  …

- Step k: Each worker who was rejected in the previous step applies to her next choice if one remains. Each firm considers the workers it has been holding together with its new applicants and tentatively assigns its seats to these workers one at a time *in the firm's preference order*. Any remaining applicants are rejected.
- Stop: The algorithm terminates when no worker is rejected, and each worker is assigned his/her final tentative assignment.

Two mathematical results regarding stability (Gale and Shapley 1962)

- Definition: A matching is “stable” if there aren’t a worker and firm, not matched to each other, who would both prefer to be.
- **Existence** theorem: A stable matching exists for every preferences of firms and workers.
- **Side optimality** theorem: When all preferences are strict, the deferred acceptance algorithm produces an optimal stable matching for the side of the market that makes proposals.

Two mathematical results regarding incentives (Roth, 1982)

- Definition: A *mechanism is stable* (for the marriage/college admissions markets) if for every preferences it produces a stable matching.
- Impossibility Theorem: No stable mechanisms exist which make it a dominant strategy for everyone to state their true preferences
- Possibility Theorem: The deferred acceptance algorithm in which the proposing side seeks only one position (e.g. workers) makes it a dominant strategy for that side to state its true preferences.

Roth, A. E. and Elliott Peranson, “The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design,” *American Economic Review*, 1999, 748-780.

One important difference from the simple models of matching is that some graduating medical students are **couples**, who must be matched to two positions.

(In the 1950’s, almost 100% of American medical grads were men, by 1970 they were 10% women, today 50%)

An initial “couples algorithm” in the 1970’s

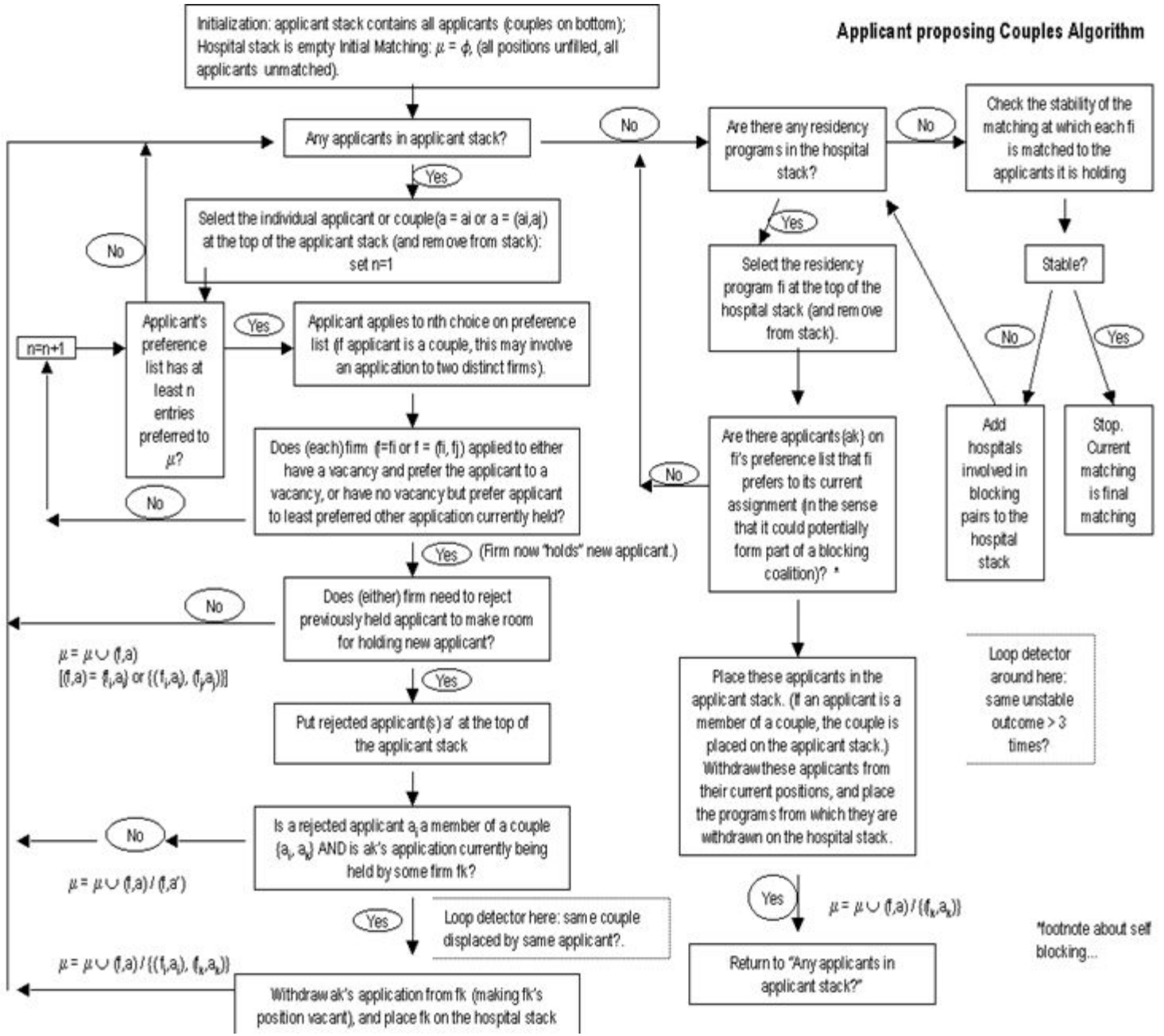
- Couples (after being certified by their dean) could register for the match as a couple.
  - They had to specify one member of the couple as the “leading member.”
  - They submitted a separate rank order list of positions for each member of the couple
- The leading member went through the match as if single.
- The other member then had his/her rank order list edited to remove positions not in the ‘same community’ as the one the leading member had matched to.

But this didn’t work well for couples

- Why?
- The iron law of marriage: You can’t be happier than your spouse.
- Couples consume *pairs* of jobs. So an algorithm that only asks for their preference orderings over *individual* jobs can’t hope to avoid instabilities (appropriately redefined to include couples’ preferences)
- But even if we ask couples for their preferences over pairs of jobs, we may still have a problem: Roth (1984) observed that the set of stable matchings may be *empty* when couples are present.

Roth, A.E. and E. Peranson, "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design," *American Economic Review*, 89, 4, September, 1999

Deferred acceptance algorithm



Roth, A.E. and Vande Vate, J.H. "Random Paths to Stability in Two-Sided Matching," *Econometrica*, 58, 1990, 1475-1480.

Theorem 2.33 (Roth and Vande Vate, 1990): (For the marriage model of 1-1 matching) Let μ be an arbitrary matching for (*M, W, **P***). Then there exists a finite sequence of matchings $\mu_1,\ldots,\mu_k$, such that $\mu = \mu_1$, μ_k is stable, and for each $i = 1,\ldots,k\text{-}1$, there is a blocking pair (m_i, w_i) for μ_i such that **μ_{i+1} follows from μ_i by satisfying the pair (m_i, w_i)**.

μ_i = {(m1,w1)…(**mi**,wi)…(mj,**wj**), …(mn,wn)}

μ_{i+1} {(m1,w1,) (wi),(**mi,wj**),(mj),…(mn,wn),)}

Donald Knuth showed that cycling is possible—welfare isn't monotone (because of the suddenly single, formerly matched people)

- This suggests a new class of algorithms, of which the deferred acceptance algorithm is a special case.
- Start with an arbitrary matching μ, and select a subset *A* of agents such that there are no blocking pairs for μ contained in *A*, and μ does not match any agent in *A* to any agent not in *A*.
- (For example, *A* could be a pair of agents matched under μ, or a single agent, or the set of all men.)
- A new player, say woman w, is selected to join *A*. If no man in *A* is part of a blocking pair with woman w, we may simply add her to *A* without changing the matching. Otherwise, select the man m whom woman *w* most prefers among those in A with whom she forms a blocking pair, and form a new matching by satisfying this blocking pair. If there is a woman *w' = μ(m)*, then she is left unmatched at this new matching, and so there may now be a blocking pair *(w',m')* contained in *A*. If so, choose the blocking pair most preferred by w' to form the next new matching.

The process continues in this way within the set $A \cup \{w\}$, like the deferred acceptance algorithm with women proposing, satisfying the blocking pairs which arise at each step until the process terminates with a matching μ_i having no blocking pairs within $A_i = S \cup \{w\}$.

The process can now be continued, with the selected set A_i growing at each stage. At each stage, the selected set has no blocking pairs in it for the associated matching μ_i, and so the process converges to a stable matching when $A_k = M \cup W$.

In the deferred acceptance algorithm with men proposing, the initial matching μ is the one at which all agents are single, and the initial set A is A=W.

In the deferred acceptance algorithm with men proposing, the welfare of the women rises monotonically throughout the algorithm. In this more general class of algorithms there is no parallel, since agents from either side may be introduced into the set A. But the set A itself grows, so the algorithm converges.

Stable Clearinghouses now using the Roth Peranson Algorithm

NRMP / SMS:

Medical Residencies in the U.S. (NRMP))

Abdominal Transplant Surgery

Child & Adolescent Psychiatry

Colon & Rectal Surgery

Combined Musculoskeletal Matching Program (CMMP)

- Hand Surgery

Medical Specialties Matching Program (MSMP)

- Cardiovascular Disease
- **Gastroenterology (1986-1999; rejoined in 2006)**
- Hematology
- Hematology/Oncology
- Infectious Disease
- Oncology
- Pulmonary and Critical Medicine
- Rheumatology

Minimally Invasive and Gastrointestinal Surgery

Obstetrics/Gynecology
Reproductive Endocrinology
Gynecologic Oncology
Maternal-Fetal Medicine
Female Pelvic Medicine & Reconstructive Surgery
Ophthalmic Plastic & Reconstructive Surgery
Pediatric Cardiology
Pediatric Critical Care Medicine
Pediatric Emergency Medicine
Pediatric Hematology/Oncology
Pediatric Rheumatology
Pediatric Surgery
Primary Care Sports Medicine
Radiology
Interventional Radiology
Neuroradiology
Pediatric Radiology
Surgical Critical Care
Thoracic Surgery
Vascular Surgery

Orthopaedic Surgery: Foot & Ankle; Sports Medicine; Trauma

Postdoctoral Dental Residencies in the United States

- Oral and Maxillofacial Surgery
- General Practice Residency
- Advanced Education in General Dentistry
- Pediatric Dentistry
- Orthodontics

Psychology Internships

Neuropsychology Residencies in the U.S. & CA

Pharmacy Practice Residencies in the U.S.

Articling Positions with Law Firms in Alberta, CA

Medical Residencies in CA (CaRMS)

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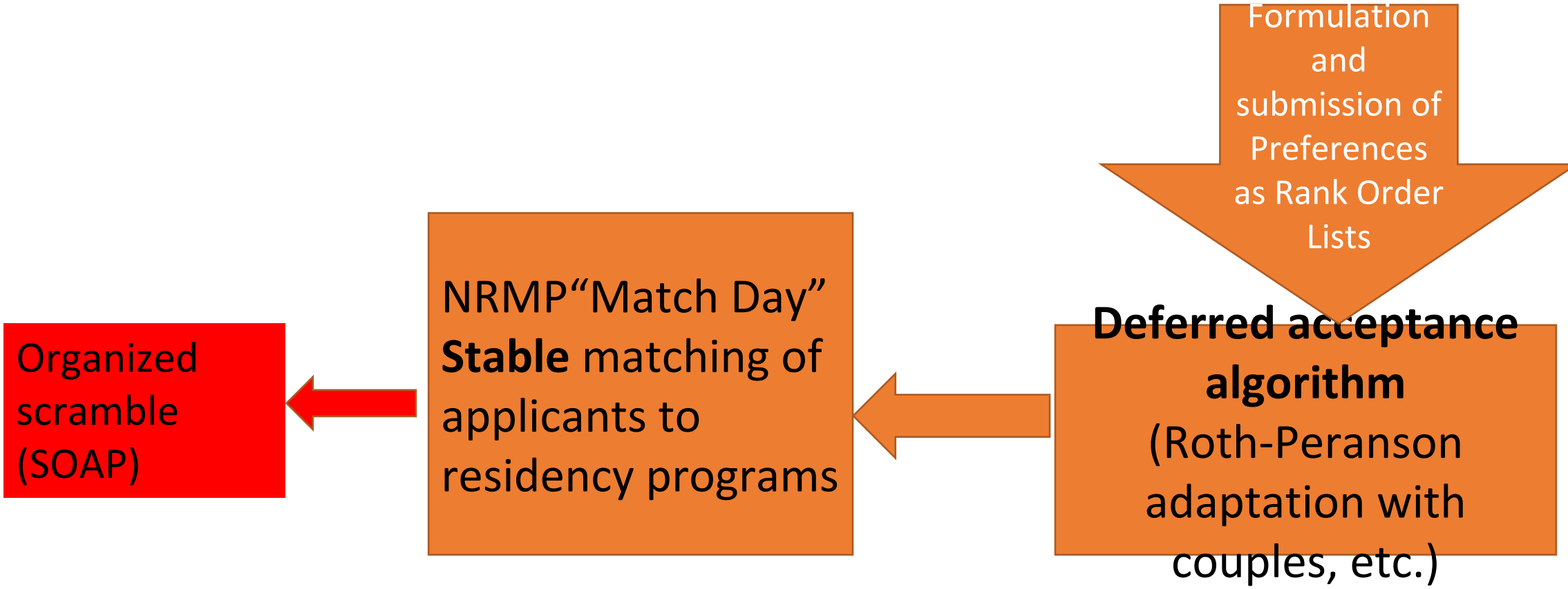
The Transition from Medical School to Residency



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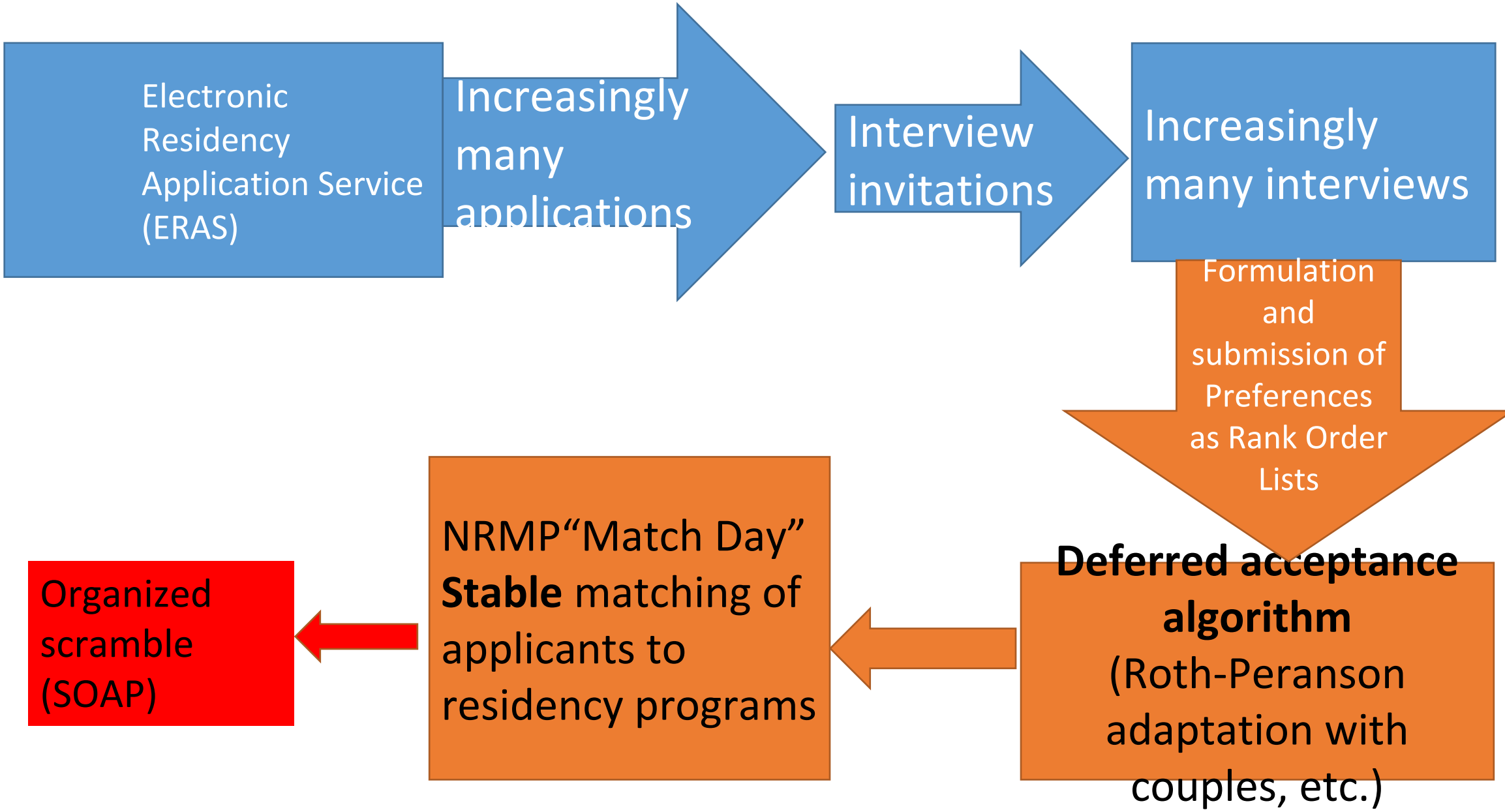

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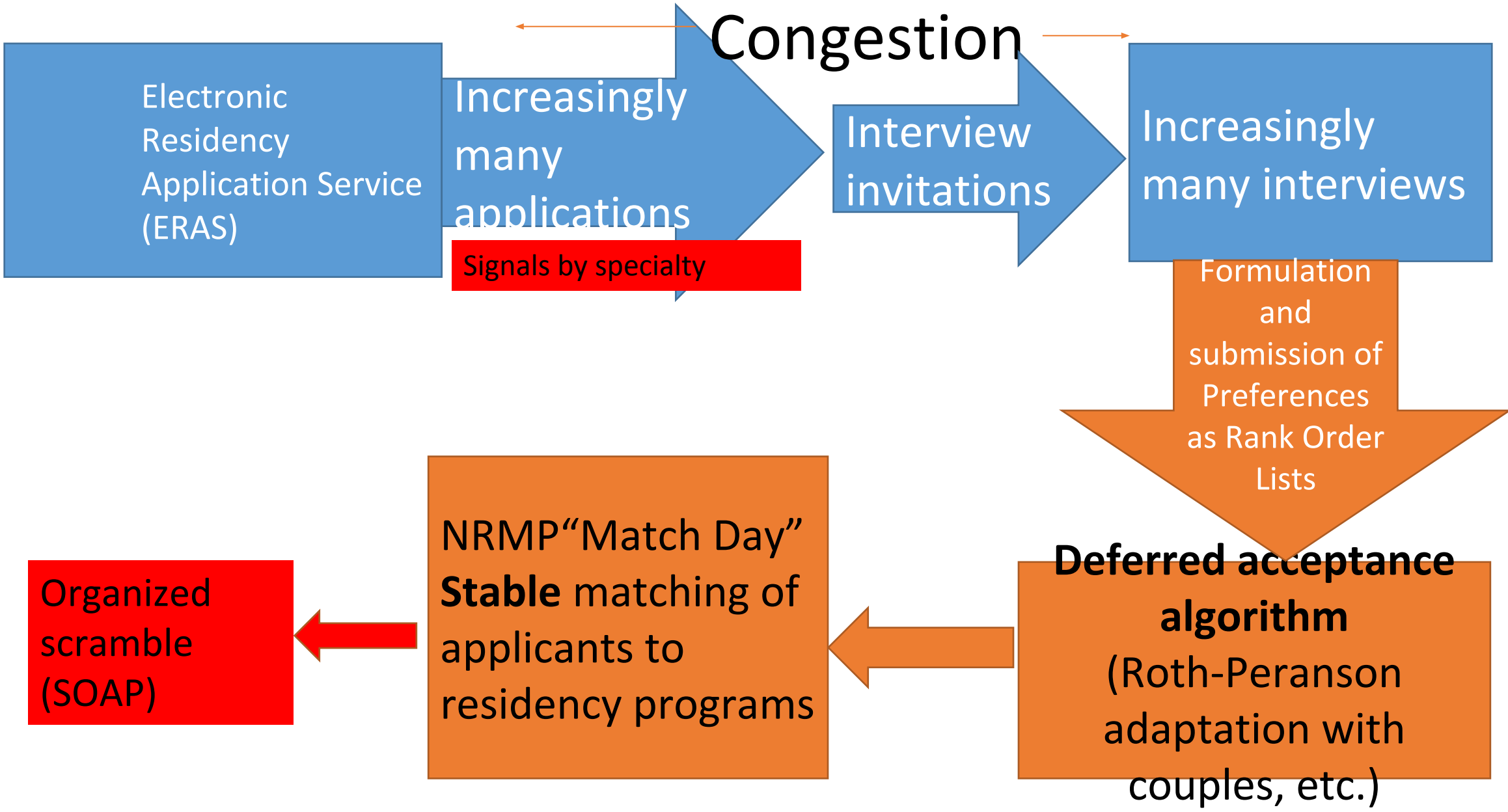


The Transition from Medical School to Residency

A symphony of suggestions in the medical literature

- Suggested solutions:
  - Signaling?
  - Interview matching?
  - Limits on applications?? On Interviews??
  - An early match (for OBGYN)???
    - (Sometimes, market design involves playing defense…)

Viewpoint

January 23, 2020

Improving the Residency Application and Selection Process

An Optional Early Result Acceptance Program

Maya M. Hammoud, MD, MBA[1]; John Andrews, MD[2]; Susan E. Skochelak, MD, MPH[2]

» Author Affiliations | Article Information

JAMA. 2020;323(6):503-504. doi:10.1001/jama.2019.21212

"One possible approach might be an **early** result acceptance program (ERAP), in which students would be permitted to apply to a maximum of 5 programs, and **programs would be limited to filling half of all their available spots**. Match results could be available several months prior to the current match, and **students who enter the ERAP and do not match could join the regular match**."

Why is an early match unstable? (It's no longer deferred acceptance)

Example: Consider one program P1, with **2 positions**.

P1 prefers applicants A1 and A2 to all other applicants

Applicants A1and A2 prefer P1 to all other programs.

- In the current NRMP Match: A1, A2 match with P1
- If P1 and A1 and A2 participate in the early match with **one position offered by P1**:
  - At most one of A1 and A2 can match early with P1, say A1.
  - If A2 matches early, she would regret not waiting for the regular match: she could have matched to the remaining position at P1.
  - In the later match, P1 fills its remaining position with an applicant it ranks lower than A2.

Observations:

- P1 forms a mutually dissatisfied pair with A2 for the combined early and regular match.
- Note that A2 and P1 do worse than in a single match.

Itai Ashlagi, Ephy Love, Jason I. Reminick, Alvin E. Roth; Early vs Single Match in the Transition to Residency: Analysis Using NRMP Data From 2014 to 2021. J Grad Med Educ 1 April 2023;

Descriptive statistics: OB/GYN

Table 1

Descriptive NRMP Match Statistics for OB/GYN Applicants and Program (2014-2021)

Statistics	2014	2015	2016	2017	2018	2019	2020	2021
Single Applicants								
Submitted ROLs	1692	1700	1623	1605	1720	1879	1842	1880
Ranked other specialties	380	367	371	331	383	408	431	414
Ranked OB/GYN and other specialties within top 5	256	252	250	216	243	266	264	247
Top choice OB/GYN	1519	1543	1480	1493	1590	1756	1714	1773
Couples								
One member ranks OB/GYN	110	126	134	149	165	153	173	156
One member ranks OB/GYN as their top choice	106	122	130	144	161	145	163	149
Both rank OB/GYN	1	5	5	5	2	3	3	3
Programs								
Programs	250	250	254	258	272	280	289	294
Positions	1263	1275	1286	1309	1357	1412	1460	1478
Average ROL length per program	58.9	62	61.4	62.3	62.1	63	62.2	65.8
Average ROL length per position	11.7	12.2	12.1	12.3	12.4	12.5	12.3	13.1

Simulation results

Value	2014	2015	2016	2017	2018	2019	2020	2021
Single applicant (top choice OB/GYN) outcomes								
More preferred, n (%)	129 (8.5)	136 (8.8)	137 (9.3)	123 (8.2)	142 (8.9)	119 (6.8)	143 (8.3)	139 (7.8)
More preferred who matched early, n	0	1	0	1	0	1	1	1
Less preferred, n (%)	227 (14.9)	226 (14.6)	242 (16.4)	232 (15.5)	229 (14.4)	250 (14.2)	239 (13.9)	256 (14.4)
Less preferred who matched early, n	166	146	183	153	164	162	160	188
Programs' outcomes, %								
More preferred	37.2	37.2	37.4	38.8	33.5	37.1	37	41.5
Less preferred	22.8	28.4	21.7	25.6	26.5	27.1	30.8	23.8
Mixed	20.8	16.4	22.8	15.5	19.9	15	11.1	14.3
Couples' outcomes, %								
One matched early	46.4	38.1	45.5	46.3	42.4	30.7	40.5	37.2
Unacceptable outcome when one matched early	35.3	31.2	27.9	23.2	21.4	19.1	12.9	22.4
Mutually dissatisfied pairs among singles with less preferred matches, %	70.9	61.1	73.1	65.9	67.7	61.6	66.9	70.3
Programs in mutually dissatisfied pairs, %	54.4	52.8	60.2	58.9	52.9	49.6	54.3	52.4
Programs with more preferred outcomes: percentage that match with an applicant in a mutually dissatisfied pair, %	91.4	78.5	83.2	83	89	80.8	81.3	75.4

In Economics, we use signaling to help deal with interview congestion (deciding whom to interview)

Coles, Peter, John H. Cawley, Phillip B. Levine, Muriel Niederle, Alvin E. Roth, and John J. Siegfried, "The Job Market for New Economists: A Market Design Perspective," *Journal of Economic Perspectives*, Fall 2010

Signaling for Interviews in the Economics Job Market

The AEA coordinates a mechanism through which applicants can signal their interest in receiving an interview at the January meetings. **In mid-November, each registered *JOE* candidate on the economics job market will have the opportunity to register and designate no more than two departments (or other employers) to whom to send a signal of particular interest.** The AEA will transmit these signals to the departments the candidates choose. (Signals will not be made public.) Employers do not need to do anything to register to receive signals; signals will be sent automatically to the email address provided at the time the *JOE* listing was submitted.

2 signals

See Signaling for Interviews in the Economics Job Market for a detailed description as well as the Terms of Use and Privacy Policy.

Please contact aea_signals "at" aeaweb.org with any questions or problems.

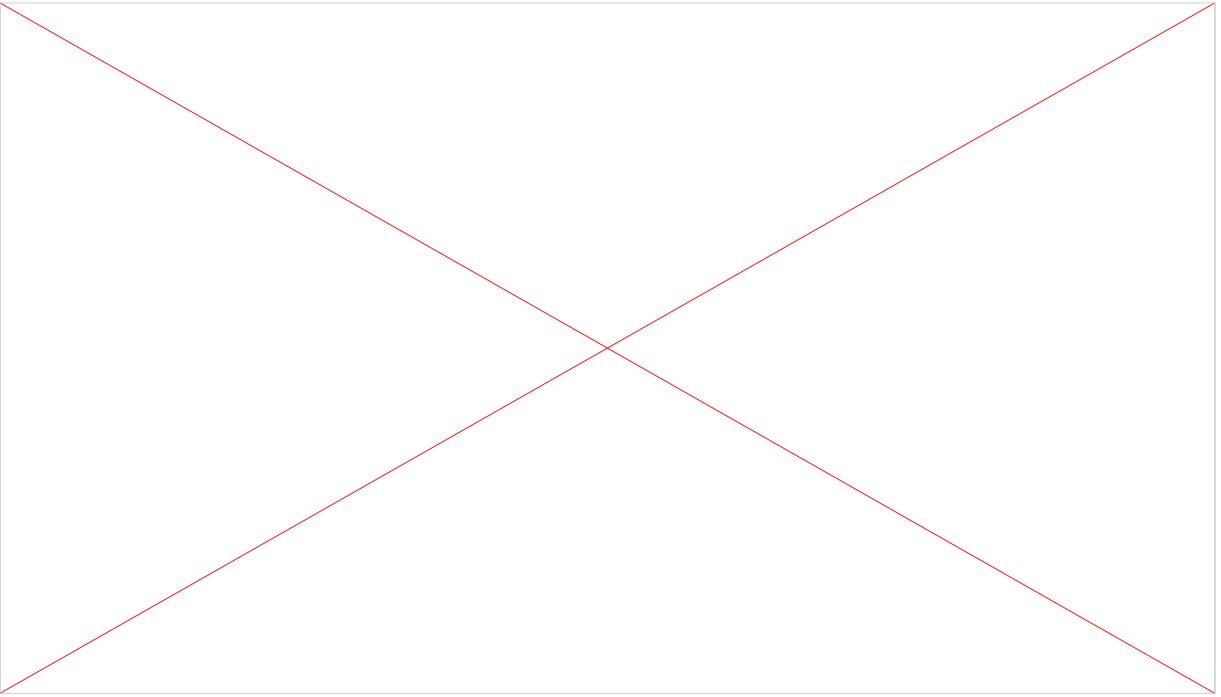
2021 Signaling Timeline

- Signaling Registration & Signal Selection will open on November 11, 2021.

- Signaling Registration & Signal Selection will close on November 29, 2021 at 5:00 p.m. EST.

- Signals will be sent to employers on December 1, 2021.

Medical specialties using signals



30 signals seems to be functioning as a “soft” cap on applications…

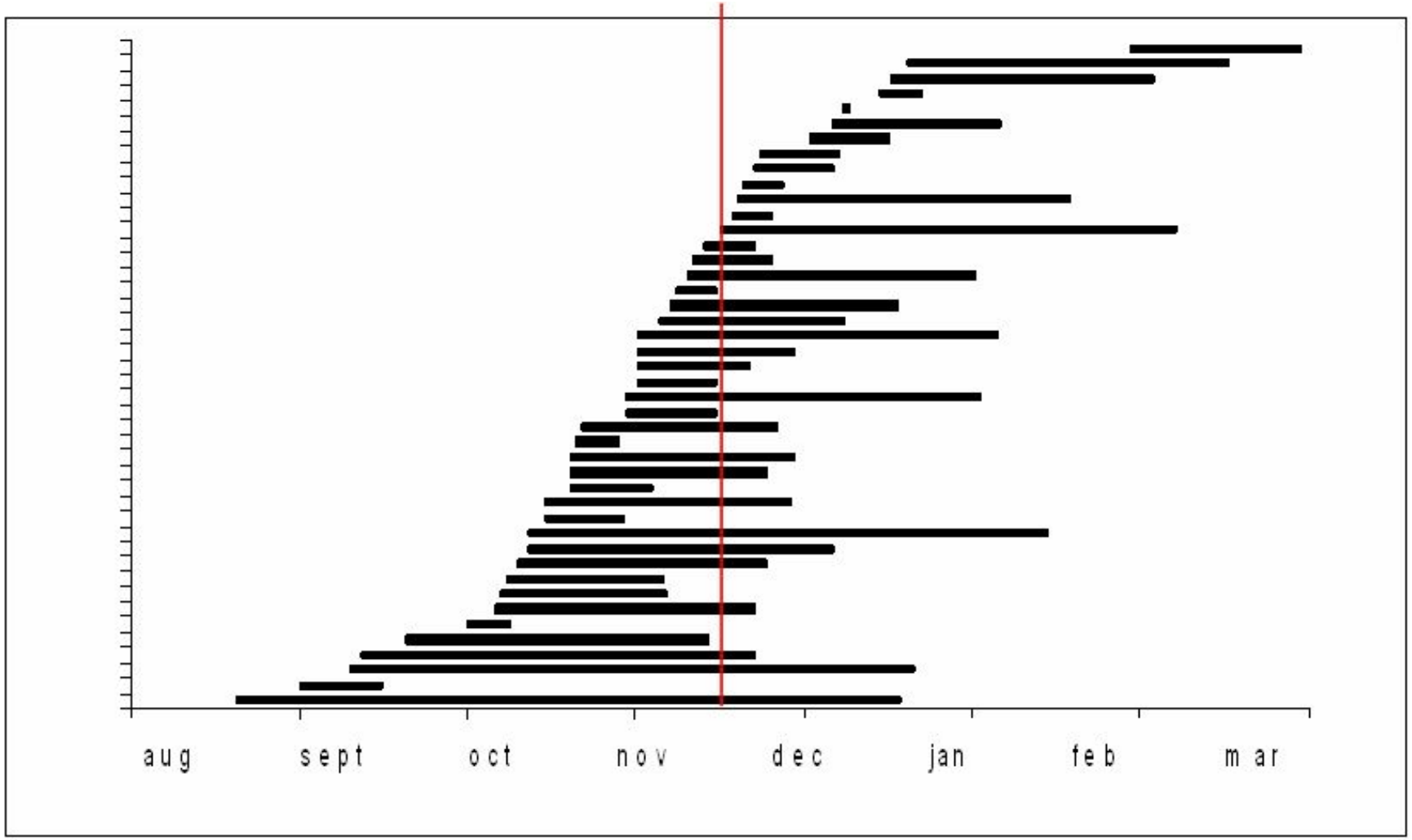
Specialty	# of signals
Adult Neurology	3
Anesthesiology	5
Dermatology	3
Radiology (diagnostic & interventional)	6
Emergency Medicine	5
General Surgery	5
Internal Medicine	7
Internal Medicine/Psychiatry	2
Neurological Surgery	8
Obstetrics & Gynecology	**3 Gold 15 Silver**
Orthopedic Surgery	**30**
Pediatrics	5
Physical Medicine & Rehabilitation	4
Preventive Medicine	3
Psychiatry	5

Reminder: What does unraveling look like?

- Gastroenterology fellowships
  - Niederle, Muriel, and Alvin E. Roth. "The gastroenterology fellowship match: how it failed and why it could succeed once again." *Gastroenterology* 127, no. 2 (2004): 658-666.

Gastroenterology: Dates during which fellowship programs were making offers. Each program is represented by one of the horizontal lines, indicating the (maximal) dates during which it could have had outstanding offers (2005 survey data, n=44).

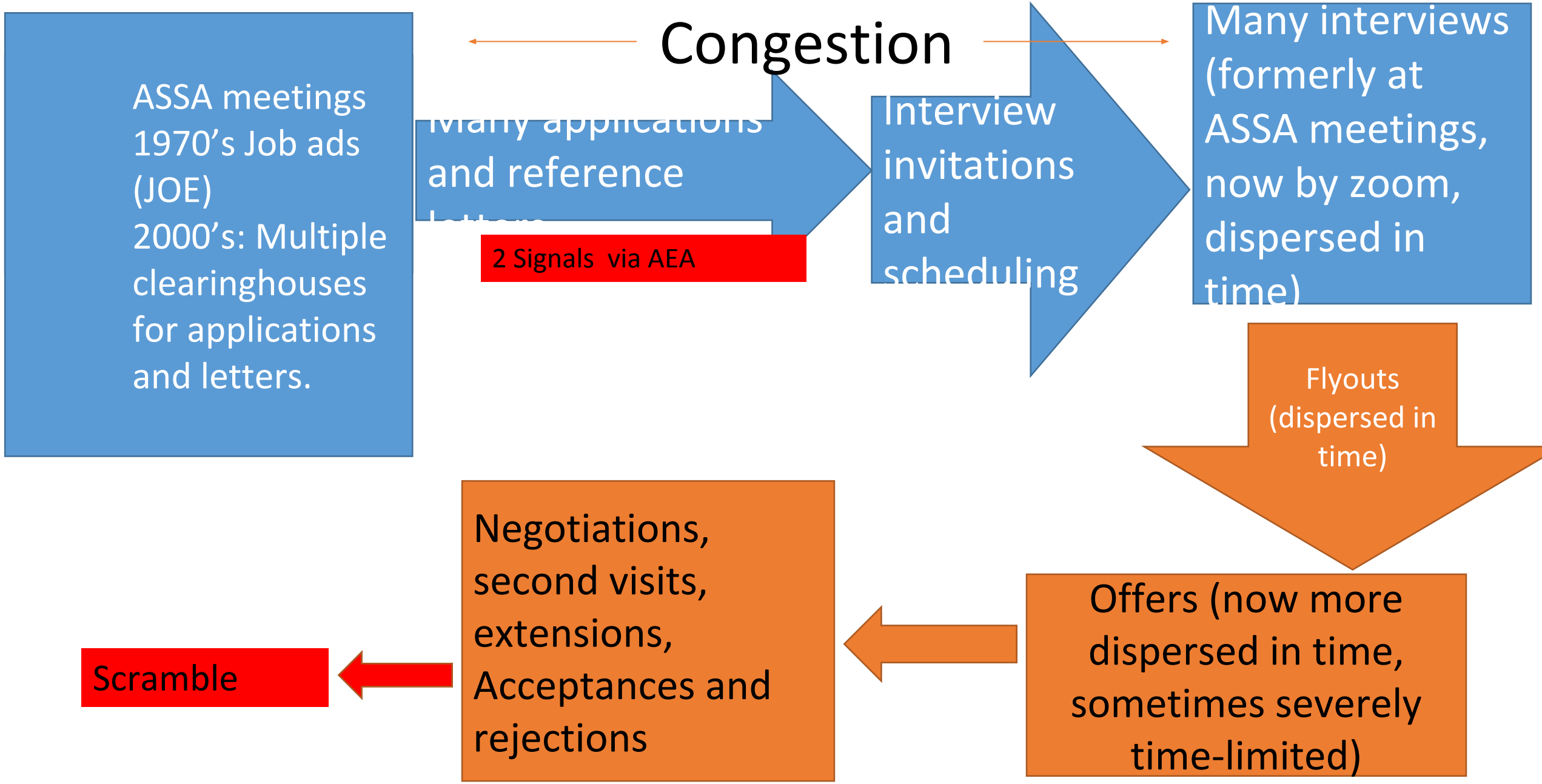
(As of November 15, 11 (27%) programs had already finished making offers, 12 (25%) had not yet started, and 21 (48%) were in the midst.)



Which markets are unraveled?

- It appears that markets in which transactions are made at early, uncoordinated times are markets in which there are both
  - Exploding offers
  - Binding commitments
- Many markets have institutions that directly address when offers can be made and accepted, and what it means for an offer to be accepted.

The Transition from Econ Grad School to post-Ph.D Job



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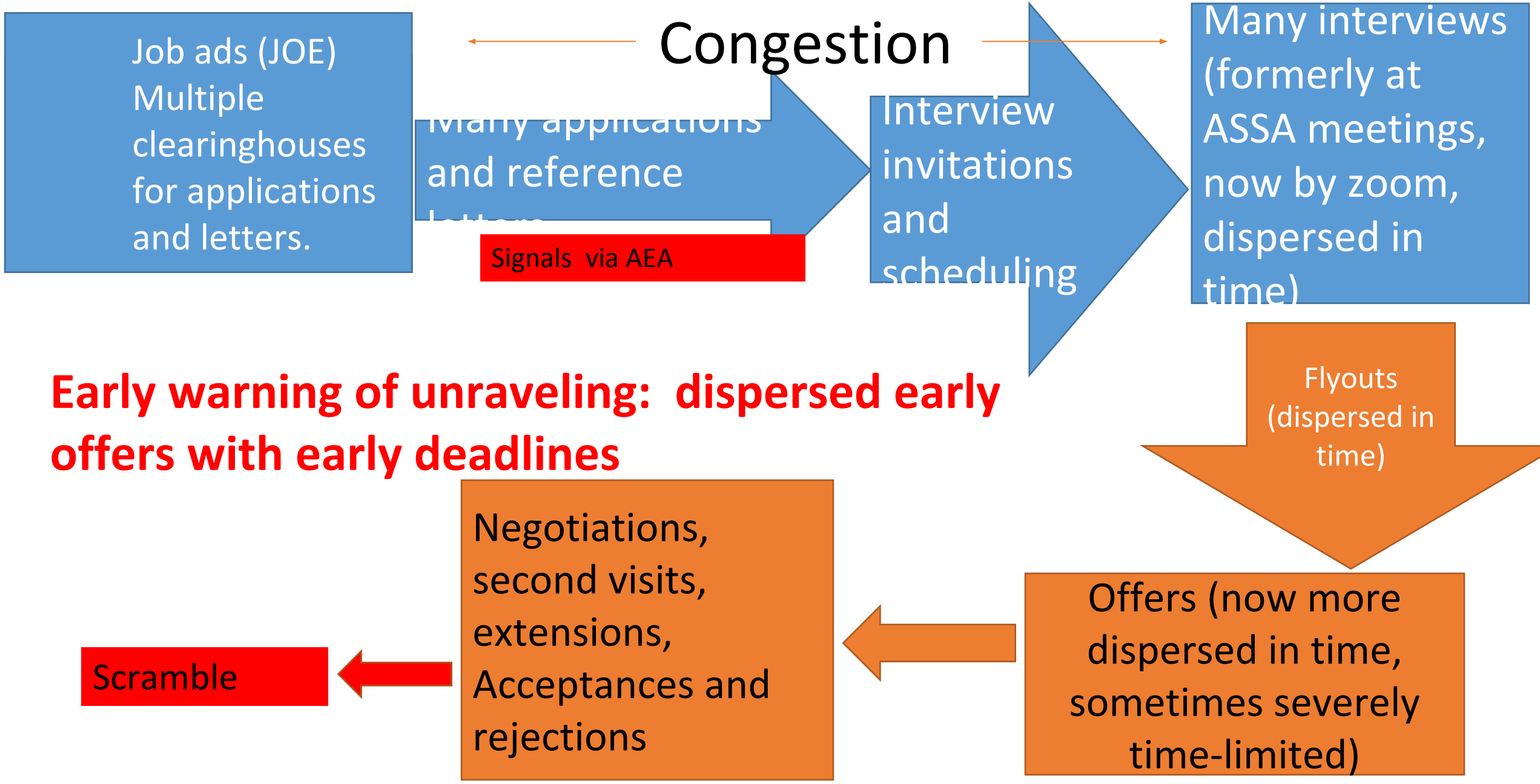

Job ads (JOE)
Multiple clearinghouses for applications and letters.
Congestion
and reference
Signals via AEA
Interview invitations and scheduling
Many interviews (formerly at ASSA meetings, now by zoom, dispersed in time)
Early warning of unraveling: dispersed early offers with early deadlines
Flyouts (dispersed in time)
Offers (now more dispersed in time, sometimes severely time-limited)
Negotiations, second visits, extensions, Acceptances and rejections
Scramble

Econ jobmarket today

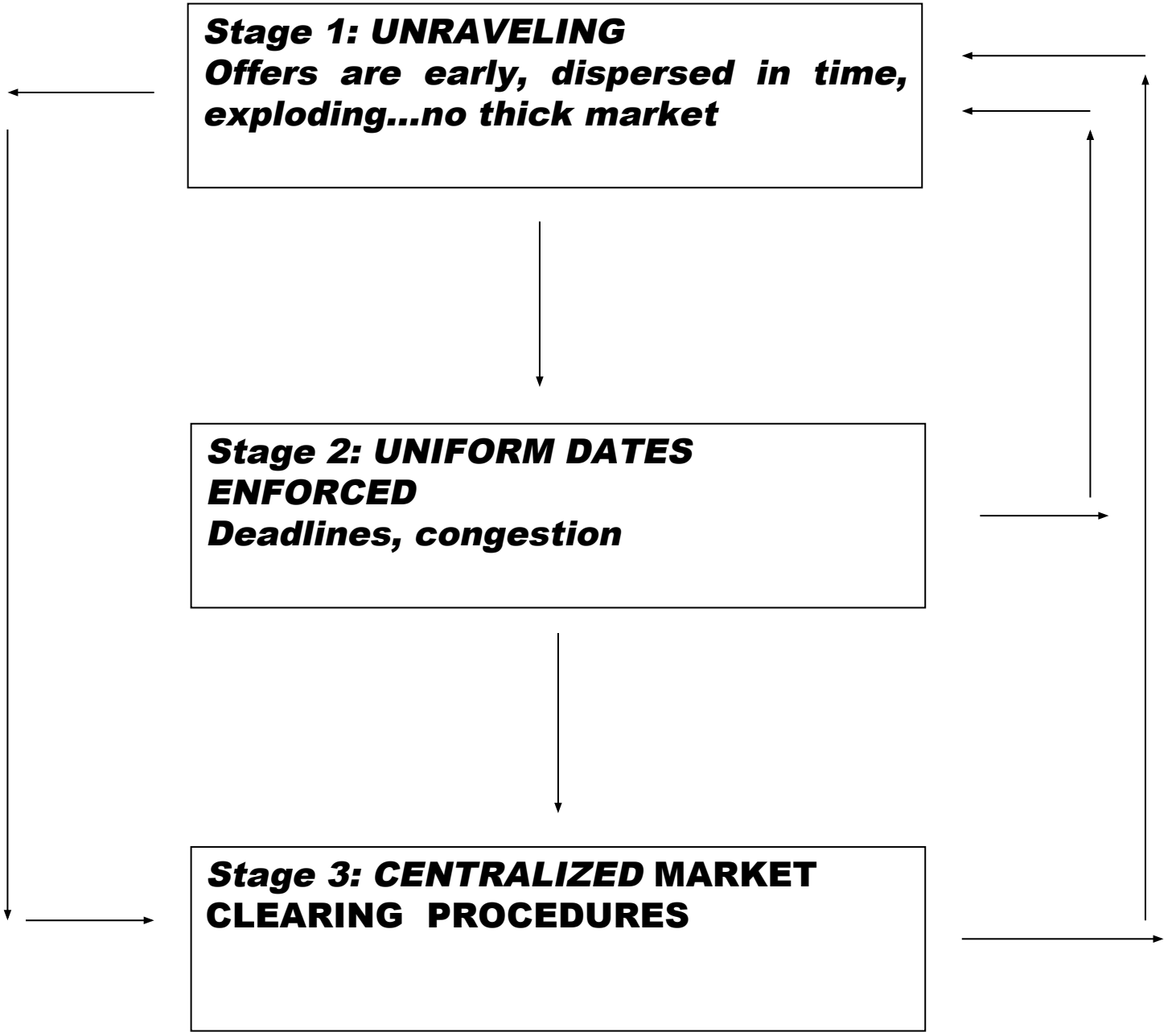
- Markets that move earlier:
  - European market, Ag-Econ, Marketing, some industry (NABE tech conf)
  - Can’t make multiple offers: LACs, Federal agencies
- Late movers
  - Elite research universities
  - Universities and colleges that don’t reconvene til late in January
- We’re in a period of disequilibrium
  - It seems likely that guidelines will be offered, and adjusted

American Finance Association guidelines

- The AFA rookie job market cycle of 2021-2022 created uncertainty, confusion, and unneeded stress for job market candidates and for recruiters. **In the interest of developing a more coordinated job market that benefits all involved, the AFA Board has the following suggested guidelines.**
- **Timing of interviews:**
- Initial interviews can be virtual or in person, but the AFA recommends that the **initial interviews should not begin before December 15, 2022, and that the timing of the "campus visit" should occur after the AFA meeting**.
- **Timing of job offers:**
- In order to facilitate the best matching between candidates and positions, **the AFA Board believes strongly that job offers should remain open until at least February 20**. The AFA Board also encourages employers to abstain from giving exploding offers with too short of a time frame, since they are unfair to the candidates. Consequently, the AFA promotes the following professional norm: **If a job candidate receives and accepts a coercive exploding offer (i.e., one that expires before February 20), the AFA does not consider such an acceptance to be binding.**

- These guidelines are designed for the AFA rookie recruiting cycle and do not pertain to recruiting cycles for other job markets such as the FMA or European job markets.

Stages and transitions observed in various markets: Reasons to worry about future unraveling…



WSJ, Aug 30, 2023: **Hectic Private-Equity Recruitment Process …**

- "Private-equity firms are recruiting workers with less and less Wall Street experience every year, hoping to beat out their competitors to hire the most impressive recent college graduates.
- **Over about 48 hours every year, hundreds of first-year investment bankers file through private-equity offices for a battery of interviews and tests, hoping to land an offer** in one of the world's most highly paid industries.
- This process, called "on-cycle" recruiting, is the traditional way that buyout firms have hired their associate ranks. **This year's recruitment process kicked off July 21—the earliest date ever—for positions starting in 2025.** Firms hired candidates who have mostly just graduated from college and are beginning two-year bank-analyst programs, making offers that kick in after their programs end."

Thanks, and welcome once again

Elements of the proof: Let μ_1 be an arbitrary (w.l.o.g individually rational) matching with blocking pair (m1,w1). Let μ_2 be the matching obtained by satisfying the blocking pair, and define the set A(1) = {m1,w1}.

Inductive assumption: Let A(q) be a subset of $M \cup W$ such that there are no blocking pairs for μ_{q+1} contained in A(q), and such that μ_{q+1} does not match any agent in A(q) to any agent outside of A(q).

Then if μ_{q+1} isn't stable, there is a blocking pair (m',w') such that at most one of m' and w' is contained in A(q). (If neither of {m',w'} is in A(q), let $A(q+1) = A(q) \cup \{m',w'\}$ and let μ_{q+2} be obtained from μ_{q+1} by satisfying the blocking pair (m',w').

Otherwise, one of the pair is in A(q), say m' (in the other case the symmetric argument will apply). Let $A(q+1) = A(q) \cup \{w'\}$. Now run the deferred acceptance algorithm, just in the set A(q+1), starting with w' proposing and continuing until a matching is reached with no blocking pairs among the members of A(q+1). The output is μ_{q+2}.

TABLE 2

Value	2014	2015	2016	2017	2018	2019	2020	2021
Single applicant (top choice OB/GYN) outcomes								
More preferred, n (%)	129 (8.5)	136 (8.8)	137 (9.3)	123 (8.2)	142 (8.9)	119 (6.8)	143 (8.3)	139 (7.8)
More preferred who matched early, n	0	1	0	1	0	1	1	1
Less preferred, n (%)	227 (14.9)	226 (14.6)	242 (16.4)	232 (15.5)	229 (14.4)	250 (14.2)	239 (13.9)	256 (14.4)
Less preferred who matched early, n	166	146	183	153	164	162	160	188
Programs' outcomes, %								
More preferred	37.2	37.2	37.4	38.8	33.5	37.1	37	41.5
Less preferred	22.8	28.4	21.7	25.6	26.5	27.1	30.8	23.8
Mixed	20.8	16.4	22.8	15.5	19.9	15	11.1	14.3
Couples' outcomes, %								
One matched early	46.4	38.1	45.5	46.3	42.4	30.7	40.5	37.2
Unacceptable outcome when one matched early	35.3	31.2	27.9	23.2	21.4	19.1	12.9	22.4
Couples in mutually dissatisfied pairs	21.8	19.8	18.7	22.8	23	14.4	15	12.2
Couples receiving more preferred	11.8	14.3	10.4	10.7	12.1	16.3	9.2	7.1
Couples receiving less preferred	27.3	25.4	28.4	31.5	27.3	20.9	25.4	19.2
Single applicants in mutually dissatisfied pairs								
Single applicants in mutually dissatisfied pairs, n (%)	205 (13.5)	162 (10.5)	203 (13.7)	187 (12.5)	182 (11.4)	187 (10.6)	183 (10.7)	205 (11.6)
Single applicants who matched early, n (%)	205 (38.4)	162 (29.9)	203 (38.1)	183 (33.9)	181 (32.4)	184 (30.5)	179 (29.7)	201 (32.2)
Single applicants who matched late, n (%)	0 (0)	0 (0)	0 (0)	4 (0.6)	1 (0.1)	3 (0.4)	4 (0.5)	4 (0.5)
Mutually dissatisfied pairs among singles with less preferred matches, %	70.9	61.1	73.1	65.9	67.7	61.6	66.9	70.3
Programs in mutually dissatisfied pairs, %	54.4	52.8	60.2	58.9	52.9	49.6	54.3	52.4
Programs with more preferred outcomes: percentage that match with an applicant in a mutually dissatisfied pair, %	91.4	78.5	83.2	83	89	80.8	81.3	75.4