STRATEGY-PROOF MECHANISMS

& INCENTIVE AUCTIONS

PAUL MILGROM

AUGUST 13, 2023

INTRODUCTION

Two Parts to this Talk

1. Formulation and General Mathematical Theory

2. Newer Theory and Its Application to "Incentive Auctions"

Discussion:

- *Centralized and Decentralized Economies*
- *Social Choice*

FORMULATION

- Social Choice Formulation
  - We start we finite sets of agents $\mathcal{N} = \{1, \dots, N\}$ and $\mathcal{K} = \{1, \dots, K\}$
  - $\mathcal{P}$ is the set of permutations of $(1, \dots, K)$
  - Each agent has a preference list $P_n \in \mathcal{P}$, with corresponding order $\succ_n$
  - Social choice mechanism: $M: \mathcal{P}^N \to \mathcal{K}$
- Strategy-Proofness (aka "Truthfulness")

STRATEGY-PROOFNESS

Definition

A social choice mechanism is **strategy-proof** (also known as **truthful**) if for every two preference profiles $P, P' \in \mathcal{P}$ and every agent $n \in \mathcal{N}$, $\mathcal{M}(P) \succcurlyeq_n \mathcal{M}(P'_n, P_{-n})$.

GIBBARD-SATTERTHWAITE THEOREM

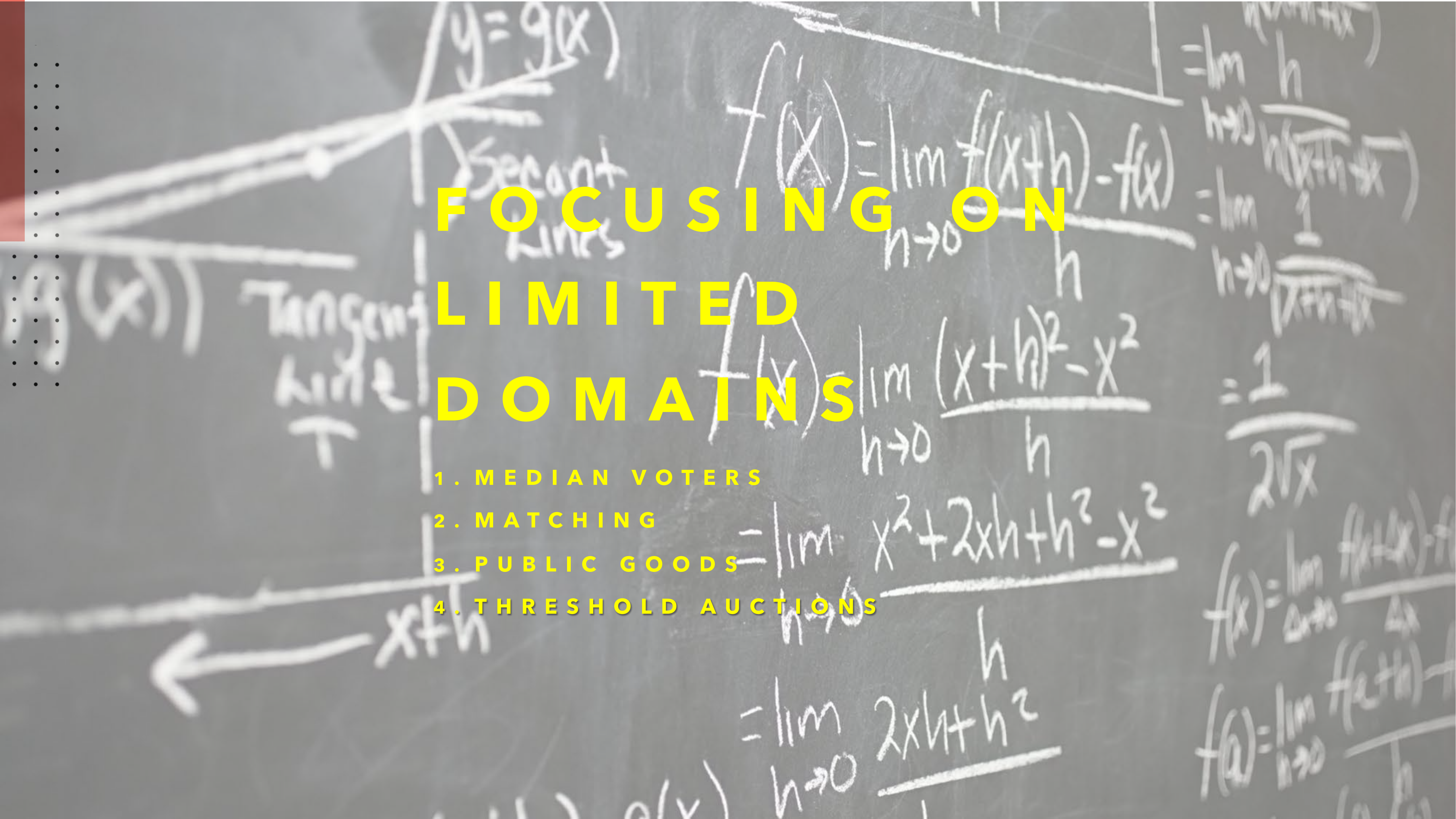
Definition

Agent n is a **dictator** for $\mathcal{M}$ if it always selects agent n's most preferred choice in its range:

For all $P \in \mathcal{P}^N$ and all $k \in \mathcal{M}(\mathcal{P}^N)$, $\mathcal{M}(P) \succcurlyeq_n k$.

Theorem

If $|\mathcal{M}(\mathcal{P}^N)| \geq 3$ and $\mathcal{M}$ is strategy-proof, then there exists a dictator n for $\mathcal{M}$.



FOCUSING ON LIMITED DOMAINS

1. MEDIAN VOTERS
2. MATCHING
3. PUBLIC GOODS
4. THRESHOLD AUCTIONS

MEDIAN VOTER THEOREM

Domain-Limiting Assumption:

Alternatives are numbered ("from left to right"). Each agent n has a favorite alternative k_n and his ranking is single-peaked, that is, for any alternatives $k > k'$

- $k_n > k \Rightarrow k \succ k'$ and
- $k_n < k' \Rightarrow k' \succ k$

Theorem ("majority voting")

If the number of voters is odd, then the mechanism in which each agent reports just her favorite alternative and and the mechanism chooses the median among them is strategy-proof.

MATCHING THEOREMS

Domain Limiting Assumptions:

1. Agents care about their own spouses, but not others' spouses
2. Agents care about their own housing, but not others' housing

Theorem

1. Gale's top-trading cycle mechanism is strategy-proof.
2. The Gale-Shapley man-proposing deferred acceptance mechanism is strategy-proof for the men (but not for women).

PUBLIC GOODS MECHANISMS

Domain Limiting Assumptions:

1. Outcomes consist of a choice $k \in \{1, \dots, K\}$ and a vector of payments to agents $p \in \mathbb{R}^N$.
2. Agent n does not care about the amounts paid to other agents:
   Agent n's value for (k, p) is $v_n(k) + p_n$

Definition

The Vickrey-Clark-Groves family of mechanisms is parameterized by a set of functions $\{h_n(v_{-n}) \to \mathbb{R}\}$. For each h, the mechanism map profiles of reported values $v = (v_1, \dots, v_N)$ into outcomes $\left(k^*(v), p^h(v)\right)$ according to

$$k^*(v) \in \arg\max_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} v_n(k)$$

$$p_n^h(v) = \sum_{m \neq n} v_m\left(k^*(v)\right) + h_n(v_{-n})$$

VCG THEOREMS

Restatement

A Vickrey-Clark-Groves (VCG) mechanism maps reported values $v = (v_1, \dots, v_N)$ into outcomes $\left(k^*(v), p^h(v)\right)$ according to

$$k^*(v) \in \arg\max_{k \in \mathcal{K}} \sum\nolimits_{n \in \mathcal{N}} v_n(k)$$

$$p_n^h(v) = \sum\nolimits_{m \neq n} v_m\left(k^*(v)\right) + h_n(v_{-n})$$

using any set of functions $\{h_n(v_{-n}) \to \mathbb{R}\}$.

Theorems

1. Every VCG mechanism is strategy-proof.
2. Every strategy-proof mechanism of the form $(k^*, \hat{p})$ is a VCG mechanism.
3. There is no VCG mechanism such that the sum of payments $\sum_n p_n^h(v)$ is a constant function.

THRESHOLD AUCTIONS

Domain Limiting Assumption:

Agent n's value is $v_n + p_n$ if she wins the item and p_n if not.

Definitions

1. An **auction mechanism** is $(\hat{k}, p)$ in which the $\hat{k}(v)$ names the identity of the single winner and only the winners pays: for any agent $n \neq \hat{k}(v)$, $p_n(v) = 0$.
2. The winner selection rule $\hat{k}$ is **monotone** if $\hat{k}(v) = n$ and $v'_n > v_n$ imply $\hat{k}(v'_n, v_{-n}) = n$.
3. A **threshold auction** is an auction mechanism $(\hat{k}, p)$ such that (i) $\hat{k}$ is monotone and (ii) $\hat{k}(v) = n \Rightarrow p_n(v) = \inf\{v'_n | \hat{k}(v'_n, v_{-n}) = n\}$.

Theorem

An auction mechanism for a single item is strategy-proof if and only if it is a threshold auction.


US BROADCAST INCENTIVE AUCTION

ECONOMIC THEORY AND ALGORITHMS

BROADCAST INCENTIVE AUCTION

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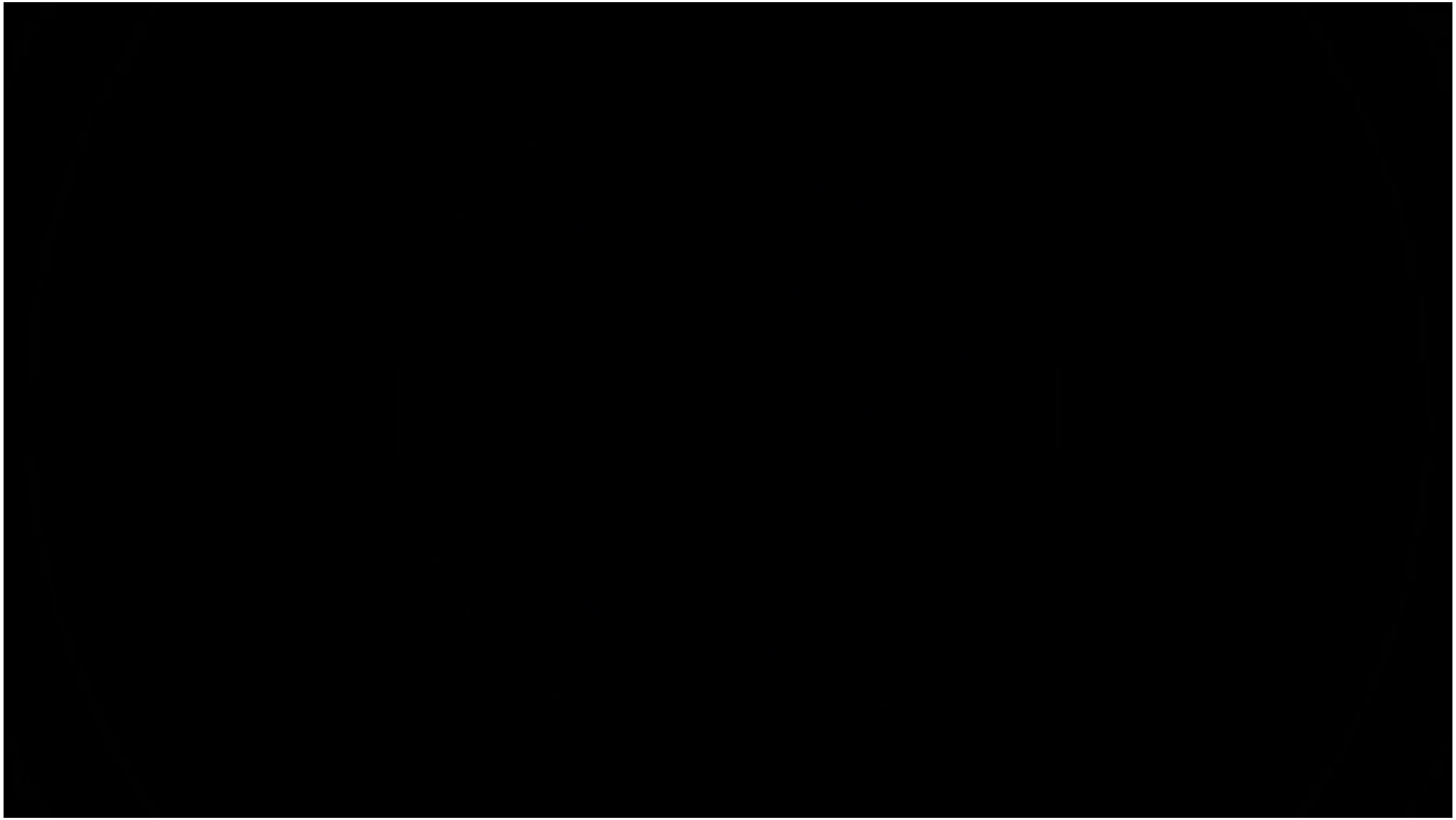
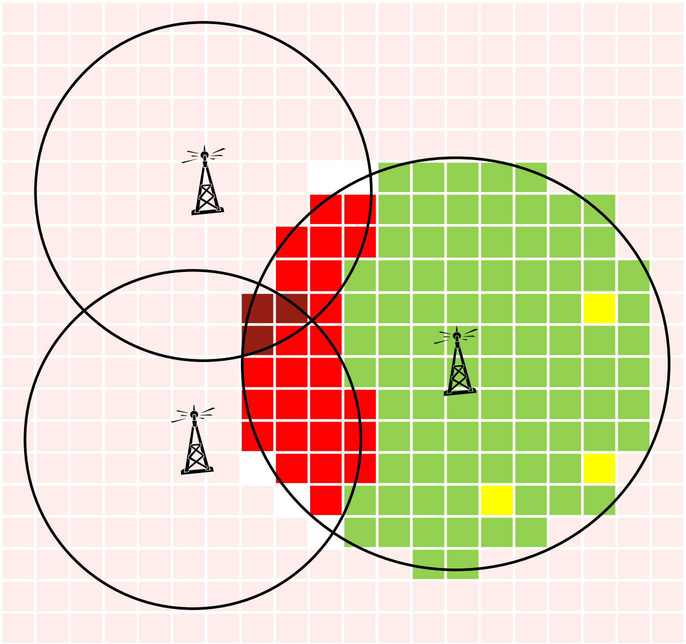


THE REVERSE AUCTION PART: WHICH STATIONS TO BUY

Goal: Given the current set of broadcasters N with station values $\{v_n\}_{n \in N}$, find the *feasible* subset of stations $S^* \in \mathcal{F}$ to broadcast in a more limited set of channels to maximize the total value of the stations remaining on-air.

$$S^* \in \arg\max_{S \in \mathcal{F}} \sum_{n \in S} v_n$$

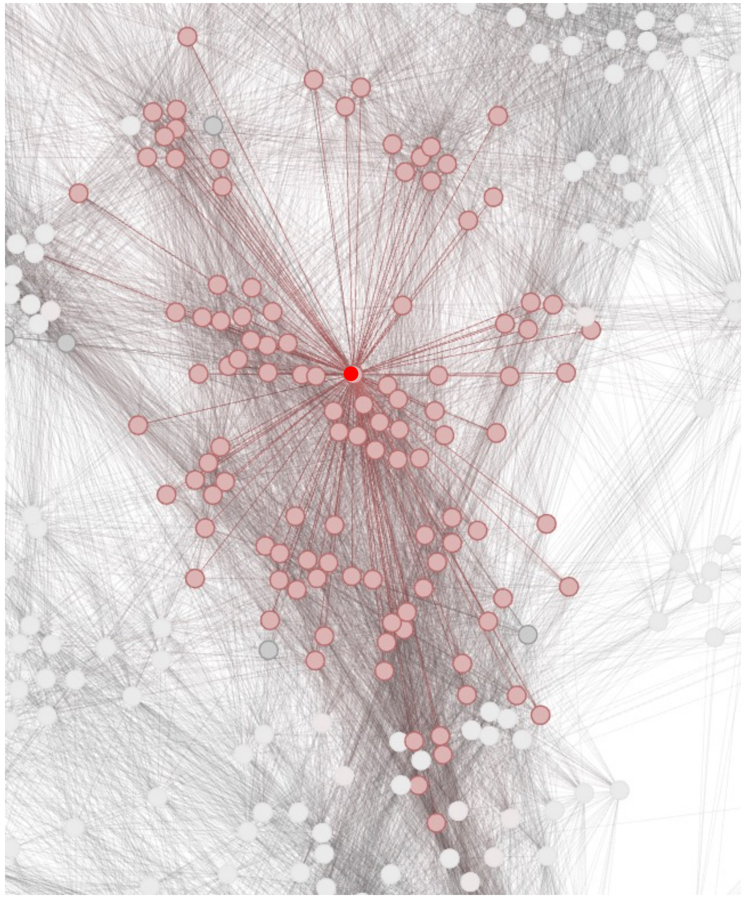
What is Feasible?



OET-69 Bulletin Coverage:
≈10 million cells (1km x 1km)

Constraint: For each station, given channel assignments, fewer than 0.5% of existing customers may suffer interference from another station.

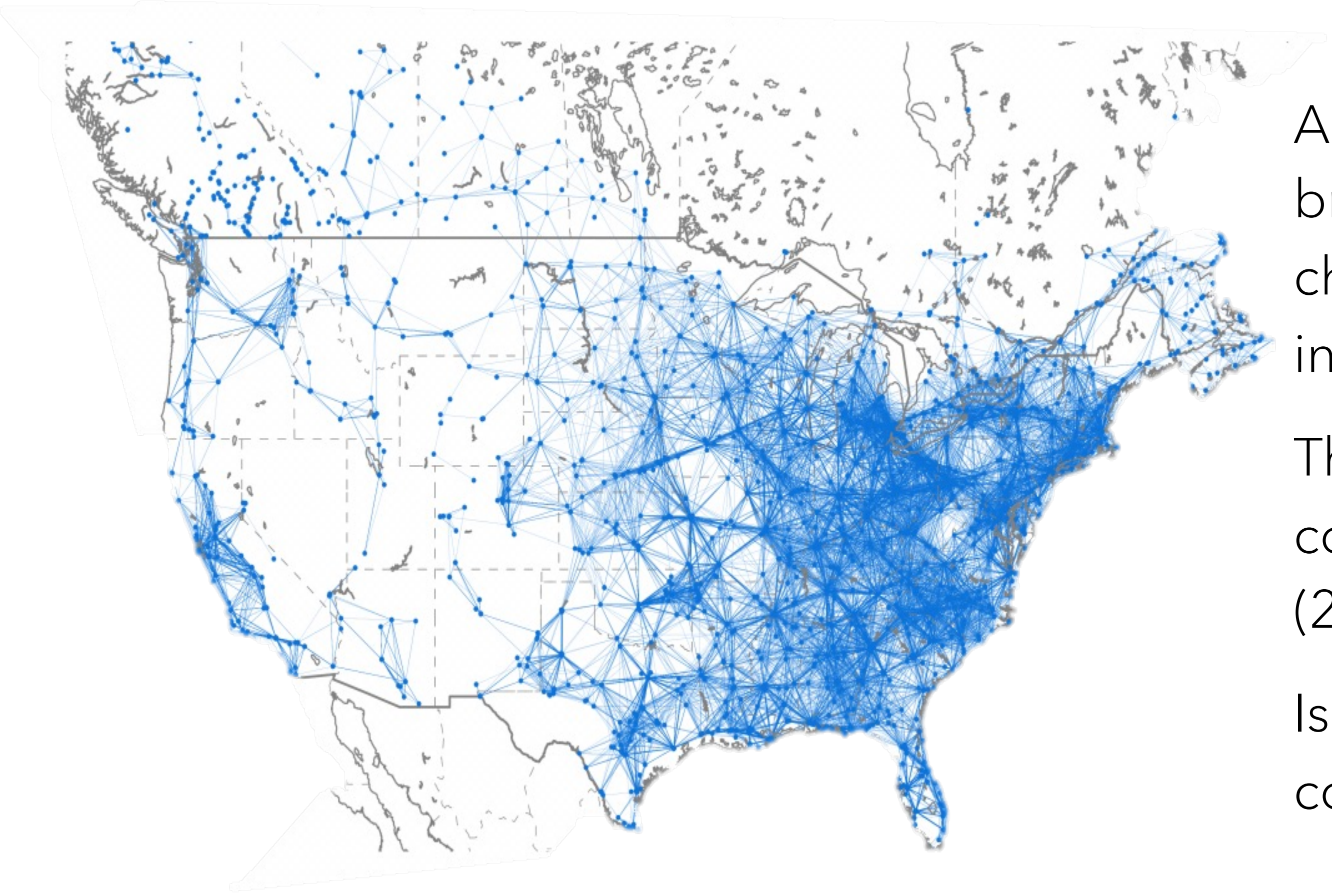
Co-channel Interference Around One Station



- Each "node" represents the location of a UHF-TV station
- Each "arc" represents a pair of stations that cannot both be assigned to the same channel without causing ("unacceptable") interference.
- Nodes connected to the central red node are shown in pink.

Most interference is co-channel interference.

Co-channel Interference Graph



A set of stations can all continue to broadcast if there is a way to assign channels to the stations without interference.

There were about 130,000 co-channel constraints as shown in the graph. (2.7 million detailed constraints)

Is there such an assignment? A graph coloring problem: it is NP-complete.

THEORY: DESCENDING CLOCK AUCTIONS

- NOTATION
  - Periods: $t = 1,2, \dots$. Active bidders at period t:$A_t \subseteq A_{t-1} \subseteq \cdots \subseteq A_1 = N$
  - Active bidders at each round can exit or not: $A_{t+1} = \{n \in A_t | n \text{ does not exit at } t + 1\}$.
  - History: $A^t = (A_1, \dots, A_t)$. Set of possible histories: H. Price function: $p: H \to \mathbb{R}_+^N$.
- RULES
  - Bidder n may exit at a round $t + 1$ if and only if and $p_{t,n}(A^t) < p_{t-1,n}(A^{t-1})$.
  - If auction ends at period T, the winners are the bidders $n \in A^T$ and they sell at prices $p_{T,n}(A^T)$
- A DESCENDING CLOCK AUCTION is a function $p: H \to \mathbb{R}_+^N$ with four properties:
  1. Only active bidders' prices can change: $n \notin A_t \Rightarrow p_{t,n}(A^t) = p_{t-1,n}(A^{t-1})$
  2. No bidder's price ever increases: $p_t(A^t) \leq p_{t-1}(A^{t-1})$.
  3. The auction is over when no price declines: $p_t(A^t) = p_{t-1}(A^{t-1}) \Longrightarrow p_{t+1}(A^{t+1}) = p_t(A^t)$
  4. The auction eventually ends: $(\exists t \geq 2)\, p_t(A^t) = p_{t-1}(A^{t-1})$

THE "*KNAPSACK PROBLEM*"

$$\max_{S \subseteq N} \sum_{n \in S} v_n \text{ subject to } \sum_{n \in S} s_n \leq K$$

Knapsack constraint

Greedy algorithm: Order items so that $\frac{v_1}{s_1} > \frac{v_2}{s_2} > \cdots$. (Ignore ties.) "Pack" items in numerical order so long as there is space remaining. If there is no room to pack an

Approximate Optimization

The difference between the maximum value and the greedy algorithm value is at most a fraction of the value of the first item m that is excluded from the knapsack, as follows:

$$v_m \frac{\left(K - \sum_{j=1}^{m-1} s_j\right)}{s_m}.$$

An equivalent descending clock auction: (Informal description)

1. The "base clock" $q\colon [0,1] \to \mathbb{R}_+$ is a continuous, decreasing function of time with $q(0)$ large and $q(1) = 0$.
2. At time $t \in [0,1]$, if there is still space for station j, its tentative offer price to go off air is $p_j(t) = s_j q(t)$.
3. Bidders choose either to "exit" (reject the offer) and or remain active in the auction.
4. After decisions at t, if there is no space left for station j, then for $t' > t$, set $p_j(t') \coloneqq p_j(t)$.
5. "Truthful bidding" means: "Remain active at t if $p_j(t) > v_j$; otherwise, exit and continue broadcasting."

Proposition (Algorithmic Equivalence)

In this clock auction, if all participants bid "truthfully," then the same bidders are packed on air in the same order as for the greedy algorithm.

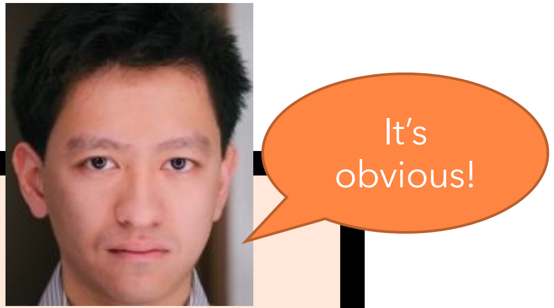
USEFUL PROPERTIES OF CLOCK AUCTIONS

ACCOMMODATES COMPUTATIONAL LIMITS

- Feasibility Guarantees
  - Reduce a station's price only if, conditional on the station deciding to exit, it is (provably) possible to assign channels to it and all other stations that have already exited.

If the proof algorithm used by the auction fails to resolve feasibility, it still always results in a feasible outcome.

Algorithm used in Incentive Auction had ~99% success rate resolving feasibility.



SOLVING THE TRUST PROBLEM

Definitions

1. A strategy σ_n for player n obviously dominates another strategy $\hat{\sigma}_n$ if for any reached information set $\mathcal{I}_n$ such that $\sigma_n(\mathcal{I}_n) \neq \hat{\sigma}_n(\mathcal{I}_n)$,

$$\max_{\sigma_{-n}:\,(\hat{\sigma}_n,\sigma_{-n})\text{ visits }\mathcal{I}_n} \pi_n(\hat{\sigma}_n, \sigma_{-n} | \mathcal{I}_n) \leq \min_{\sigma_{-n}:\,(\sigma_n,\sigma_{-n})\text{ visits }\mathcal{I}_n} \pi_n(\sigma_n, \sigma_{-n} | \mathcal{I}_n).$$

2. A strategy σ_n is obviously dominant if it obviously dominates every other strategy $\hat{\sigma}_n \in S_n \backslash \{\sigma_n\}$.

Theorem

In every descending clock auction, truthful bidding is an obviously dominant strategy.

GROUP STRATEGY-PROOFNESS

Corollary

In *every descending clock auction, no coalition has a deviation from truthful bidding that strictly increases all coalition member payoffs, regardless of any strategies of the other bidders.*

Proof: …

BUDGET CONSTRAINTS

Definition

p_B is a budget-respecting extension of p for budget B if

1. p_B is a clock descending auction for which the total cost can never exceed B
2. for any value profile v such that p realizes total cost less than B, the courses of prices for p_B and p are identical.

Theorem

For every descending clock auction p and every budget $B > 0$, there exists a budget-respecting extension p_B for budget B.

WINNER PRIVACY

Definition

An (extensive-form) communication protocol satisfies unconditional winner privacy (UWP) if no winner reveals any information about his value beyond what is needed to prove that he should win, given others' values.

Theorem

1. Every descending clock auction with truthful bidding satisfies UWP.
2. If a monotonic winner selection rule can be implemented by a protocol that satisfies UWP, then it can be implemented by a descending clock auction with truthful bidding.

"COMPETITIVE" PRICES

Notation & Observation

1. Given descending clock auction $\hat{p}$ and any value profile v, let $\hat{\omega}(v)$ denote the set of winners and $p_{\hat{\omega}}(v)$ denote the the prices they pay.
2. For losing bidders, final prices are (approximately) equal to their values.

Theorem

The price profile $\left(p_{\hat{\omega}}(v), v_{-\hat{\omega}(v)}\right)$ (in which winners bid their $\hat{p}$-prices and losers bid their values) is a full-information Nash equilibrium of the first-price auction sealed-bid auction with winner selection rule $\hat{\omega}$.

Expected Cost-Minimization

With independent values, expected costs can be more nicely expressed using "virtual costs"!

Given a procurement auction that is *truthful* and *pays zero to losing bidders* in the independent private values model, the *expected total payments* are equal to expected sum of "*virtual costs*" of the stations that are purchased:

$$\mathbb{E}[Total\ Payments] = \mathbb{E}\left[\sum_{\substack{n \in T(v) \\ T(v)^C \in \mathcal{F}}} C_n(v_n)\right]$$

- n is the bidder index,
- v is the profile of values (v_n *is* bidder n's value), and
- $T(v)$ is the set of stations taken off air
- $C_n(v_n) = v_n + \frac{F_n(v_n)}{f_n(v_n)}$
- F_n, f_n are the cdf and density for v_n.

An Auction for Greedy Cost Minimization

$$\max_{S \in \mathcal{F}} \sum\nolimits_{n \in S} C_n\,(v_n)$$

S is the set of stations left *on-air.*
Buy the rights of stations $n \in S^C$.

Theorem (Greedy Optimization)

Given the value distributions F_n, suppose that each $C_n(\cdot)$ is increasing and continuous and that station "sizes" are $s_n > 0$. If a clock auction sets prices for feasible stations at each time t to satisfy

$$p_n^*(t) = s_n C_n^{-1}\big((1-t)\overline{v}\big),$$

then, for all $v \in [0, \overline{v}]^N$, the clock auction leads to the greedy solution of the the problem of maximizing total virtual cost for the stations that remain on-air/ minimizing the total for the stations going off-air.

PROPERTIES

Descending Clock Auctions are…	✓ adaptable to *limited computation capaci*ty
	✓ obviously strategy-proof
	✓ group strategy-proof (absent transfers)
	✓ extensible to satisfy budget constraints
	✓ (uniquely) winner-privacy preserving
	✓ accommodating of various objectives

FROM RADIO SPECTRUM TO WATER

BILEN ASSAYAS, BILLY FERGUSON,
PAUL MILGROM AND BUZZ THOMPSON

APRIL 2023

"The potential impacts of low runoff conditions in the winter… and the remainder of the interim period (prior to January 1, 2027) pose unacceptable risks to routine operations of Glen Canyon and Hoover Dams; therefore, modified operating guidelines need to be expeditiously developed. "

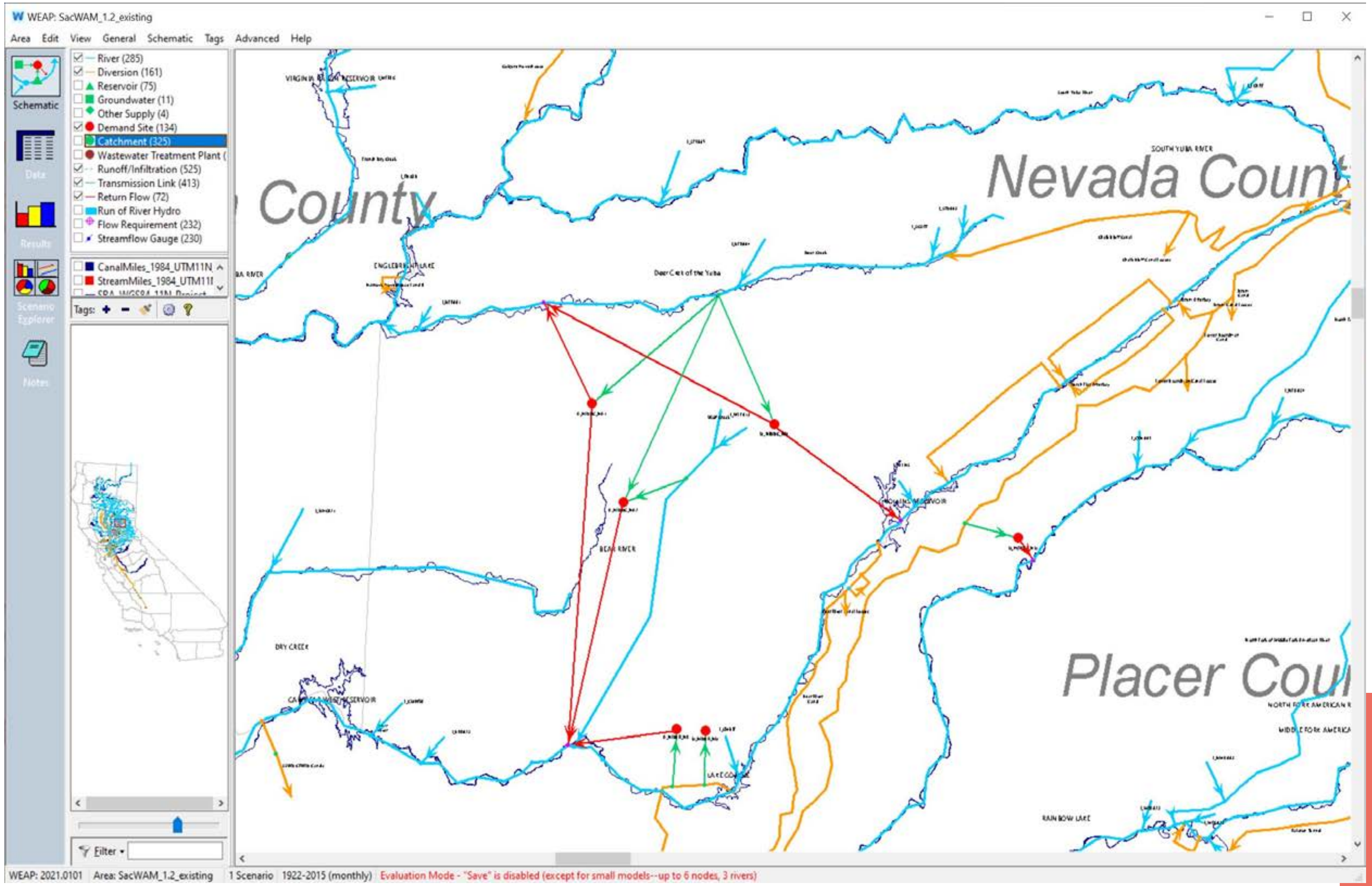
“The scope and technical complexity of issues concerning water resources management are unequalled by virtually any other type of activity presented to the courts.”

California Supreme Court

Environmental Defense Fund vs East Bay Municipal Utility District, 20 Cal.3d 327, 344 (1977).

RETURN FLOWS

- Example of water rights and hydrological network near Sacramento.
- Red dots indicate right holders.
- Green arrows indicate diversions from streams/canals.
- Red arrows indicate return flow back into the streams/canals.
- Users may return water to their original stream of diversion *and to other streams, too.*



WATER INCENTIVE AUCTION

- Water trading is difficult because of property rights
  - Unmeasured return flows
  - Heterogeneous priorities
  - Heterogeneous restrictions on use
  - Poor information
- Reconceive the broadcast incentive auction problem as voluntary procedure to change property rights (from broadcast friendly to broadband friendly) to enable efficient trade.

END