



From Contexts to Competencies A K-12 to Post-Secondary Viewpoint Critical Issues in Mathematics Education



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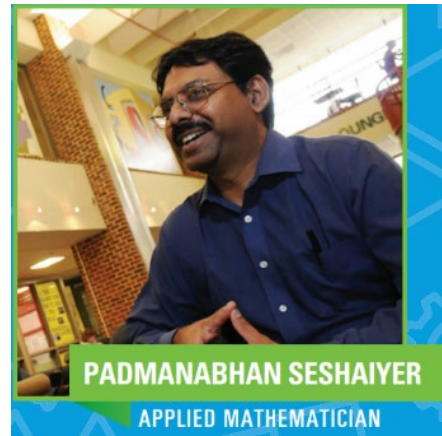
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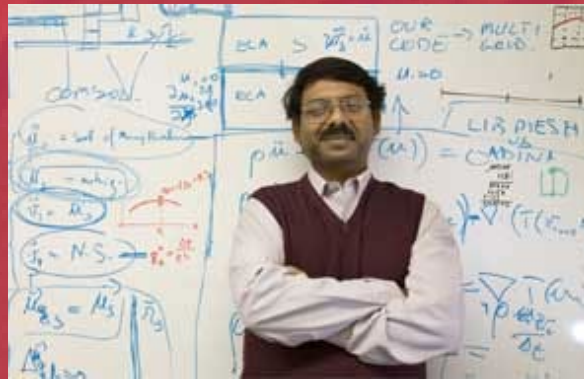
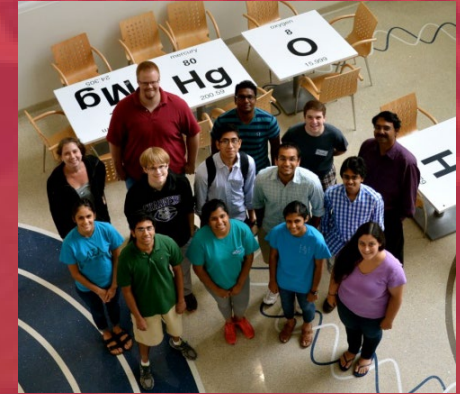
Twitter: [@padhuseshaiyer](https://twitter.com/padhuseshaiyer)



MSRI
Critical Issues in Mathematics Workshop
April 24, 2020

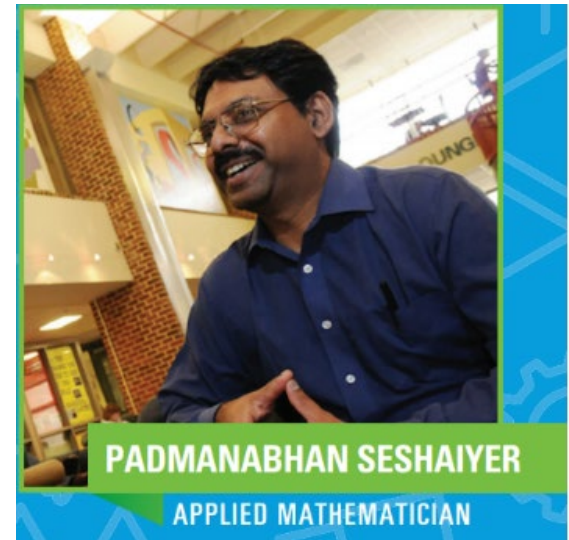


EDUCATION FOR ALL

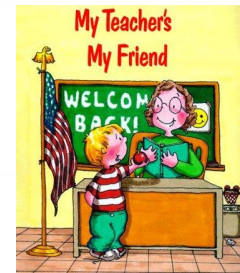
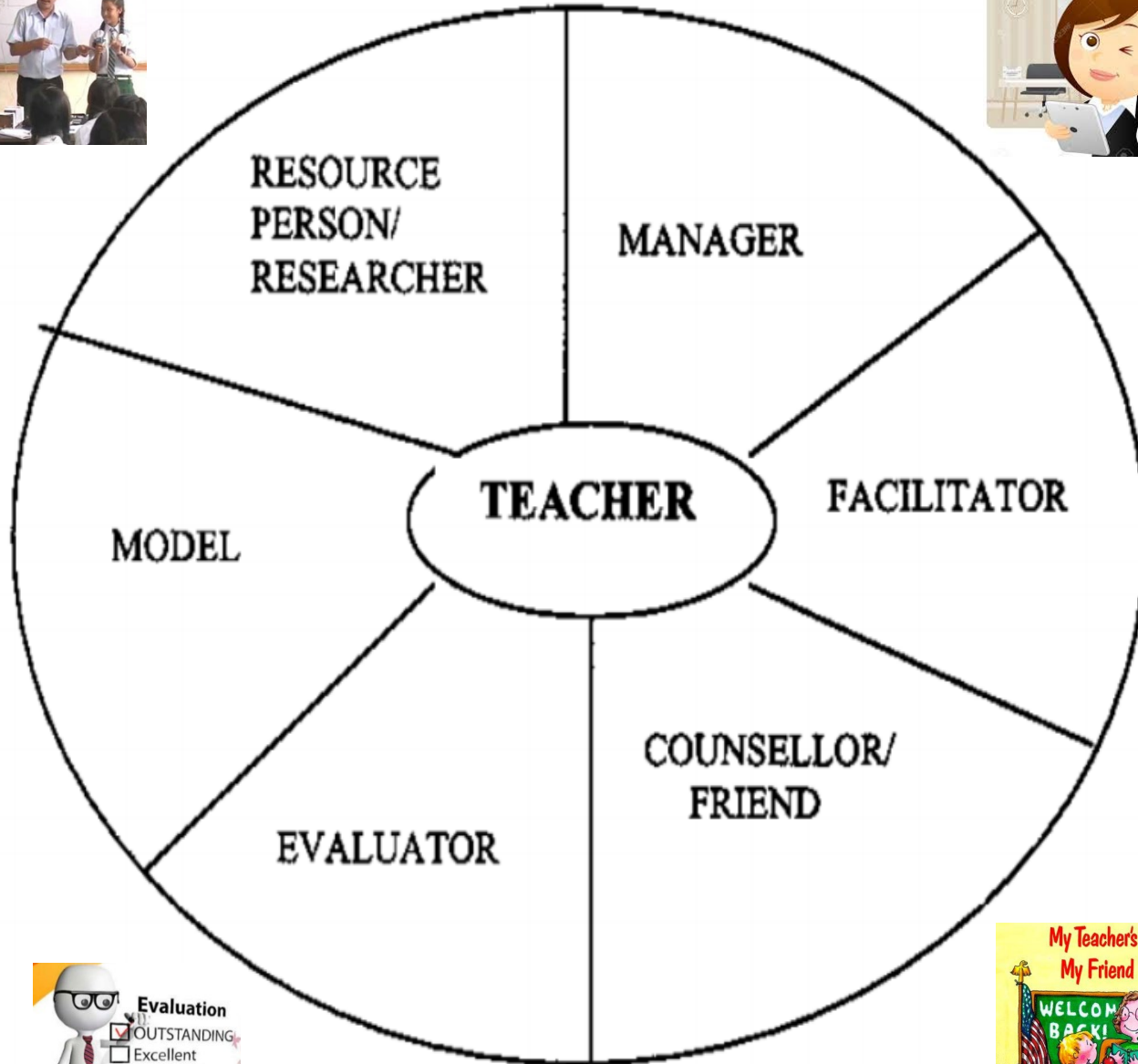


The **M** in **STEM**

TEDx



<https://www.youtube.com/watch?v=Ybxnfv203k4>






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What are some of the
fundamental competencies
needed for mathematical
problem solving?

 Respond at **PolEv.com/pseshaiyer217**

 Text **PSESHAIYER217** to **37607**
once to join, then text your message

What are some of the fundamental competencies needed for mathematical problem solving?





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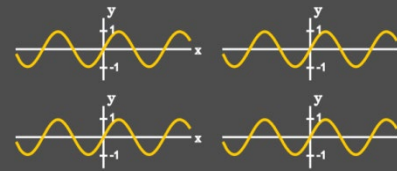
**T
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Movie Math Quiz

How many of the 16 movies can you find?

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$



$$\frac{1}{n} \sum_{i=1}^n$$

1609.344 METRES

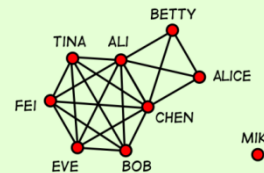
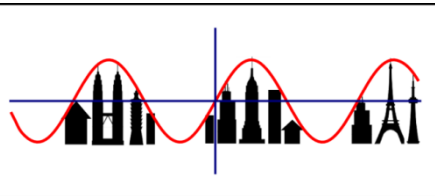
$$a+bi$$

$$F = \{x : x \text{ is a fear}\}$$

$$\sum_{x \in F} x$$

[13]

$$x \vee \{\text{cist}\}$$



$$(2i + 1, 2j + 1)$$

$$e^{i\pi} + 1 = 0$$

and
6 6 6

$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$$

$$\left| \frac{ds}{dt} \right|$$

2.7182818284590452...

Movie Math Quiz

How many of the 16 movies can you find?

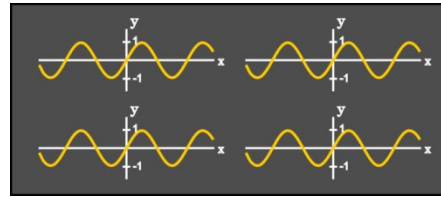
MATRIX

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

AMERICAN PI



THE SIGNS



MEAN GIRLS

$$\frac{1}{n} \sum_{i=1}^n \text{girl}_i$$

GREEN MILE

1609.344 METRES

GOLDEN EYE

$$a+bi$$

SUM OF ALL FEARS

$$F = \{x : x \text{ is a fear}\}$$
$$\sum_{x \in F} x$$

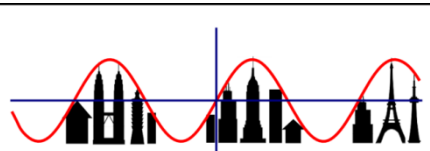
13TH FLOOR

[13]

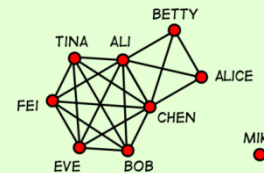
EXORCIST

$$x \vee \{\text{cist}\}$$

SIN CITY



SOCIAL NETWORK



ODD COUPLE

$$(2i + 1, 2j + 1)$$

BEAUTY & THE BEAST

$$e^{i\pi} + 1 = 0$$

and
6 6 6

HEAT

$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$$

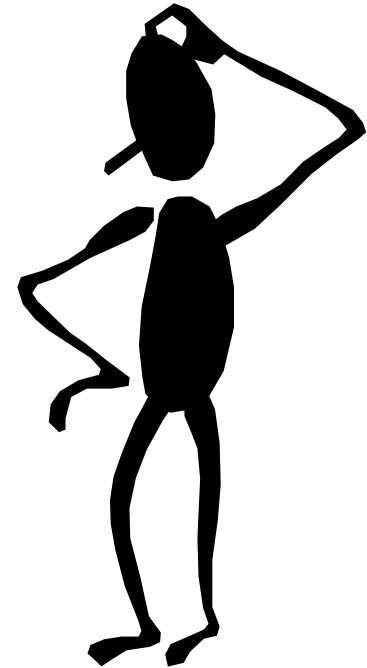
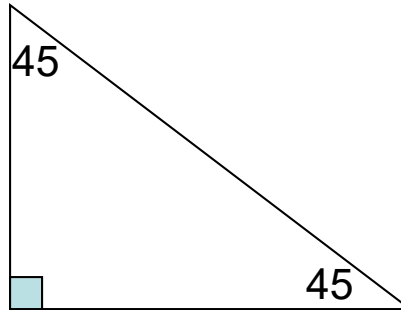
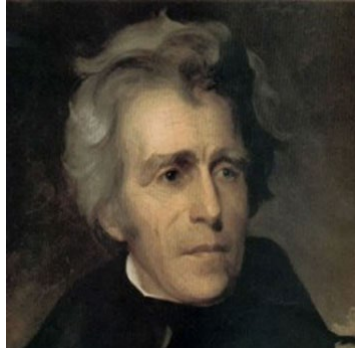
SPEED

$$\left| \frac{ds}{dt} \right|$$

WALL-E

2.7182818284590452...

What is “e” ?



2.7182818284590452



Fail early and often

FAIL
[F] **F**IRST
[A] **A**TTEMPT
[I] **I**N
[L] **L**EARNING

**“I have not failed,
I’ve just found
10,000 ways that
won’t work”**

— Thomas Edison
Inventor, Scientist



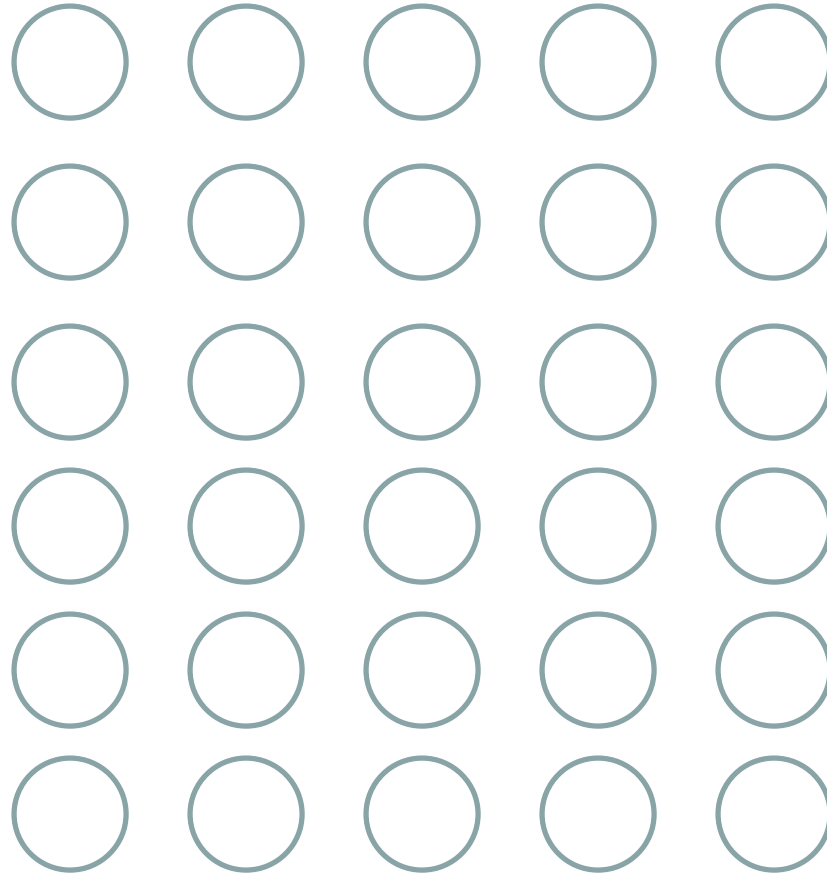


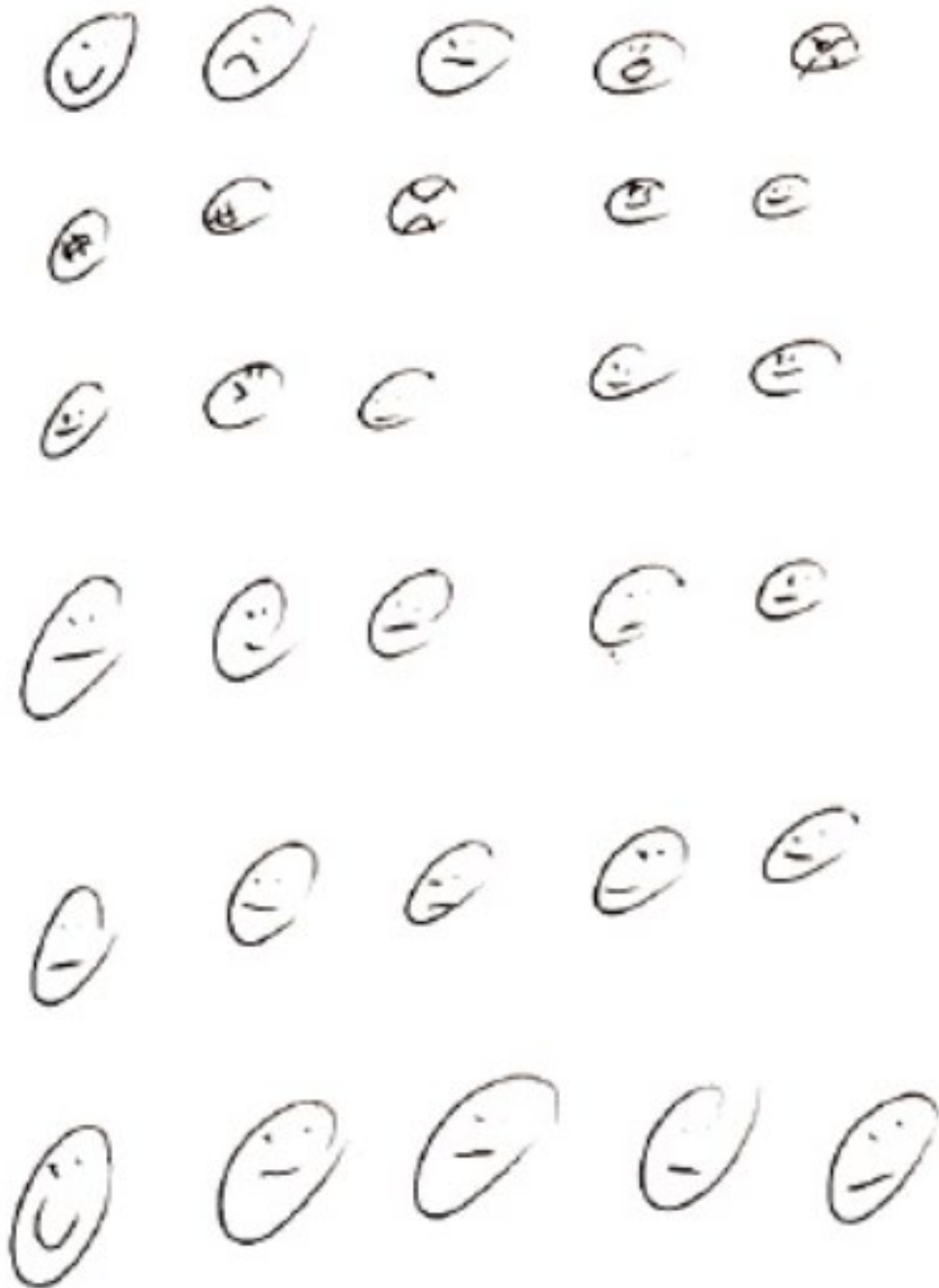
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Use each circle and create an idea.
Generate as many as you can (1 min)





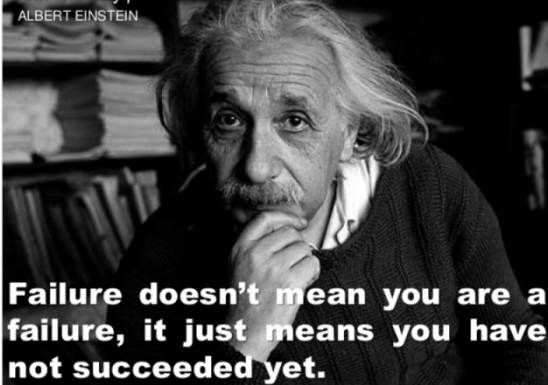
Fluency

Fail Early and Fail Often

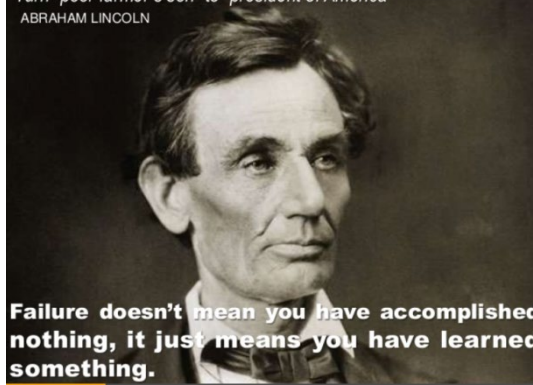
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— Thomas Edison
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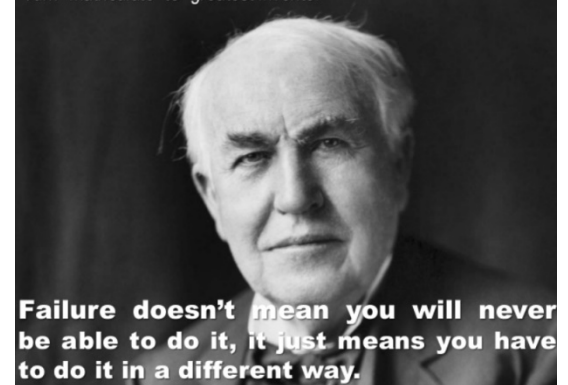
Turn “ordinary patent clerk” to “world famous scientist”
ALBERT EINSTEIN



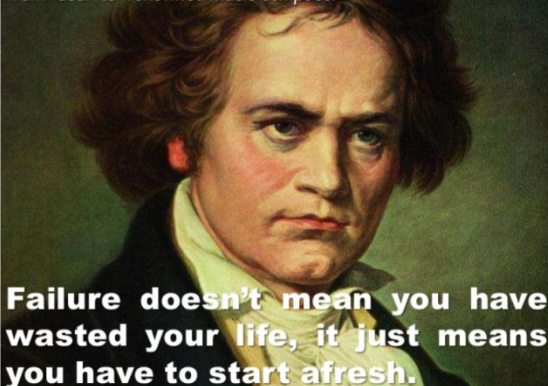
Turn “poor farmer's son” to “president of America”
ABRAHAM LINCOLN



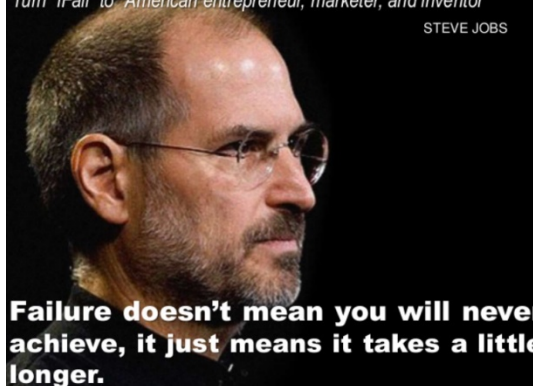
Turn “matriculate” to “greatest inventor” THOMAS EDISON



Turn “deaf” to “renowned music composer” BEETHOVEN



Turn “iFail” to “American entrepreneur, marketer, and inventor”
STEVE JOBS



Turn “early university-leaver” to “the world richest businessman”
BILL GATES





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What do you NOTICE?



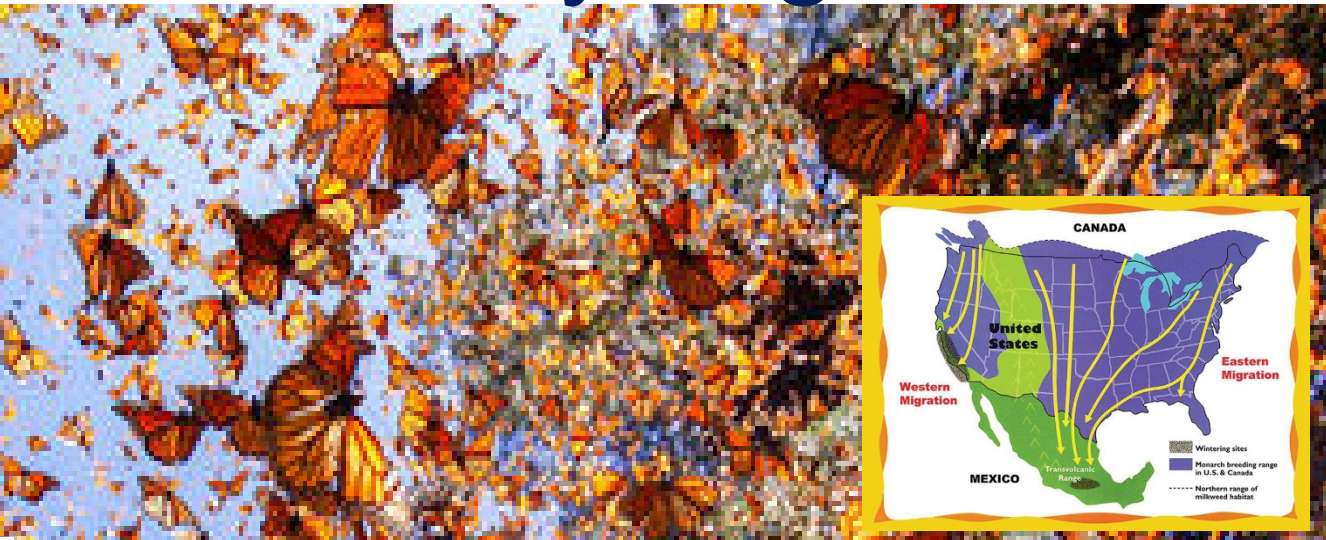
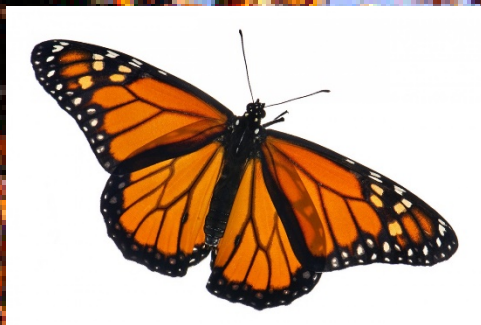
- Not symmetric
- Grouping
- Orange wings
- Color patterns
- Beauty
- Wings like tiger
- Boundaries

What do you **WONDER?**



- Defense mechanism
- Where is it going
- Speed
- Fractal
- Alive
- Body weight

Monarch Butterfly Migration



Butterfly Life Cycle Coloring Sheet

Cady the caterpillar admired adult butterflies, especially her friend Midge the Monarch. Cady wanted to know how high she would be able to fly. She knew that the flower Midge was in was 4 inches away from her and that the angle from her to the flower was 30 degrees.

She used trig to figure it out! She found that $\tan(30^\circ) = 2.3$ in. So her friend is 2.3 inches high

A few minutes later, Cady saw her friend Midge had landed in a tree. She knew the tree was 2 feet away from her and it was 10 ft tall. She wanted to know the angle from her to the tree.

So she used trig! Cady found that the $\tan^{-1}(10/2) = 72.7^\circ$ so she figured that the angle from her to Midge was 72.7 degrees

Cady wanted to know the linear speed of the bike that Midge was riding. Cady knew the wheel had a radius of 6 inches and rotated at a rate of 150 revolutions per minute

Cady found the linear speed using the formula $v = r\omega$. She multiplied $(2\pi)(6\text{in})(150)$ and got an answer of 5652in/min

Cady wanted to know

Funny, Cady was able to be like her friend Midge and she became a beautiful monarch butterfly

The End

All About Monarchs!

Monarch Butterfly

FROM YOUR OWN Monarch Butterfly

Monarch Maze



***BESTREAMING
Education for all***



BE-STREAMING

Suriname
South America

STEM
STEAM
STREAM

Padhu Seshaiyer



[https://www.youtube.com/watch?v= uXf1sKiros&feature=youtu.be](https://www.youtube.com/watch?v=uXf1sKiros&feature=youtu.be)

We STREAM Movies

We STREAM Audios

We STREAM Videos

*Let's start **STREAMING** Education!*

Try Typing this in Google

The Google logo is displayed in its standard multi-colored font: blue 'G', red 'o', yellow 'o', blue 'g', green 'l', and red 'e'.

education makes me



Google Search

I'm Feeling Lucky

Here is what you will see...



education makes me |



education makes me **depressed**

education makes me

education makes me **high**

Google Search

I'm Feeling Lucky

Report inappropriate predictions
[Learn more](#)

PARADIGM SHIFT

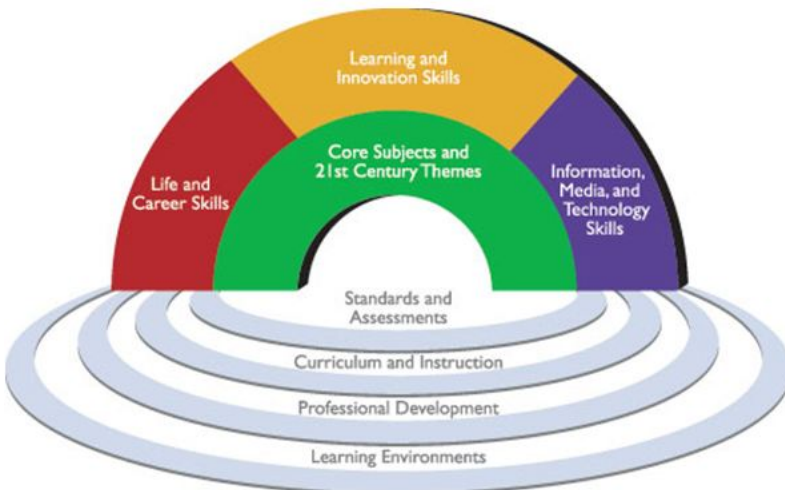


Students as Consumers

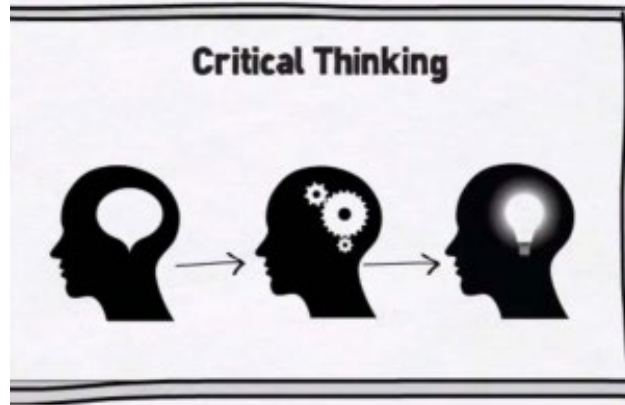
Students as Producers

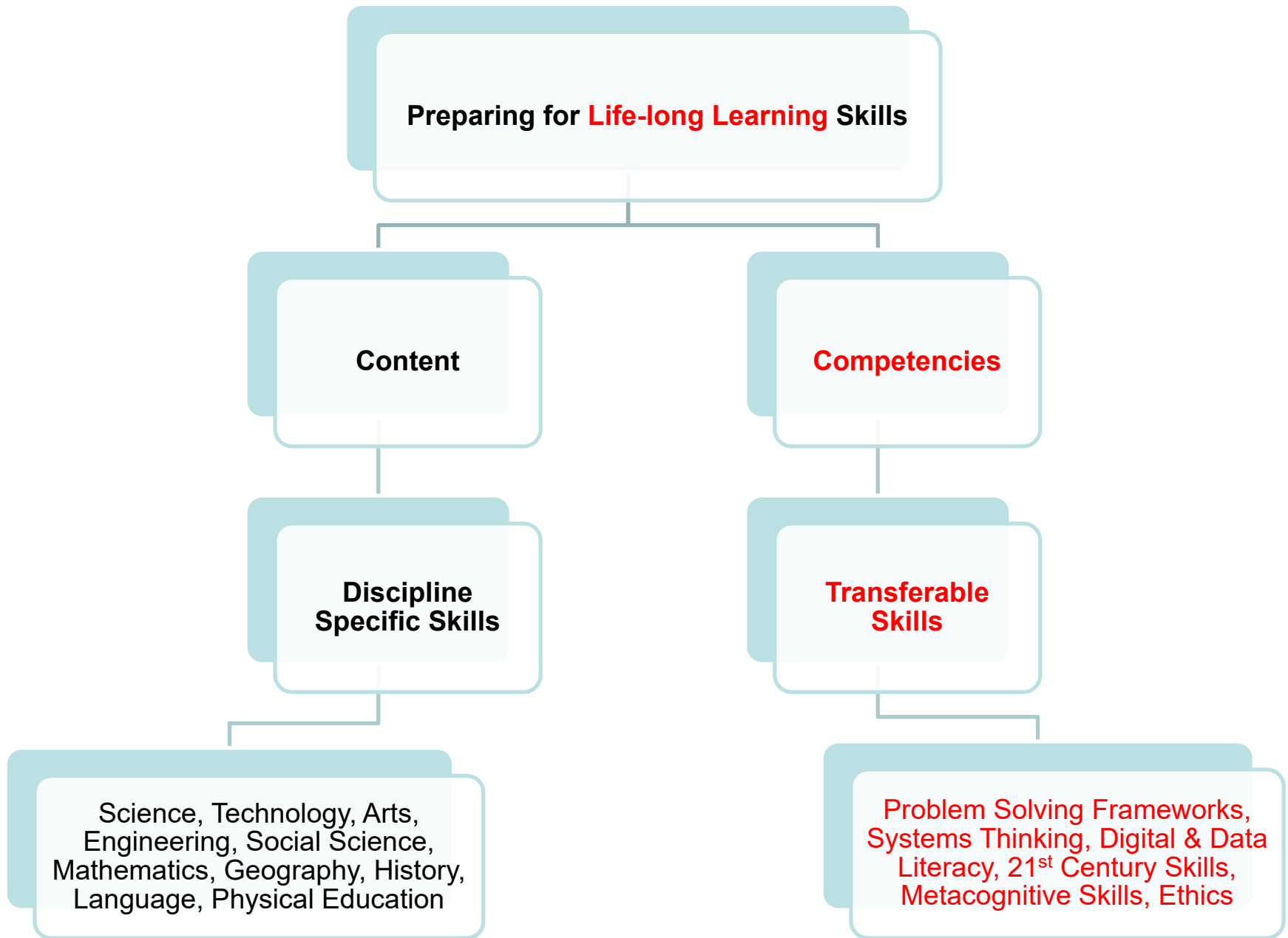


Students as peer-reviewers

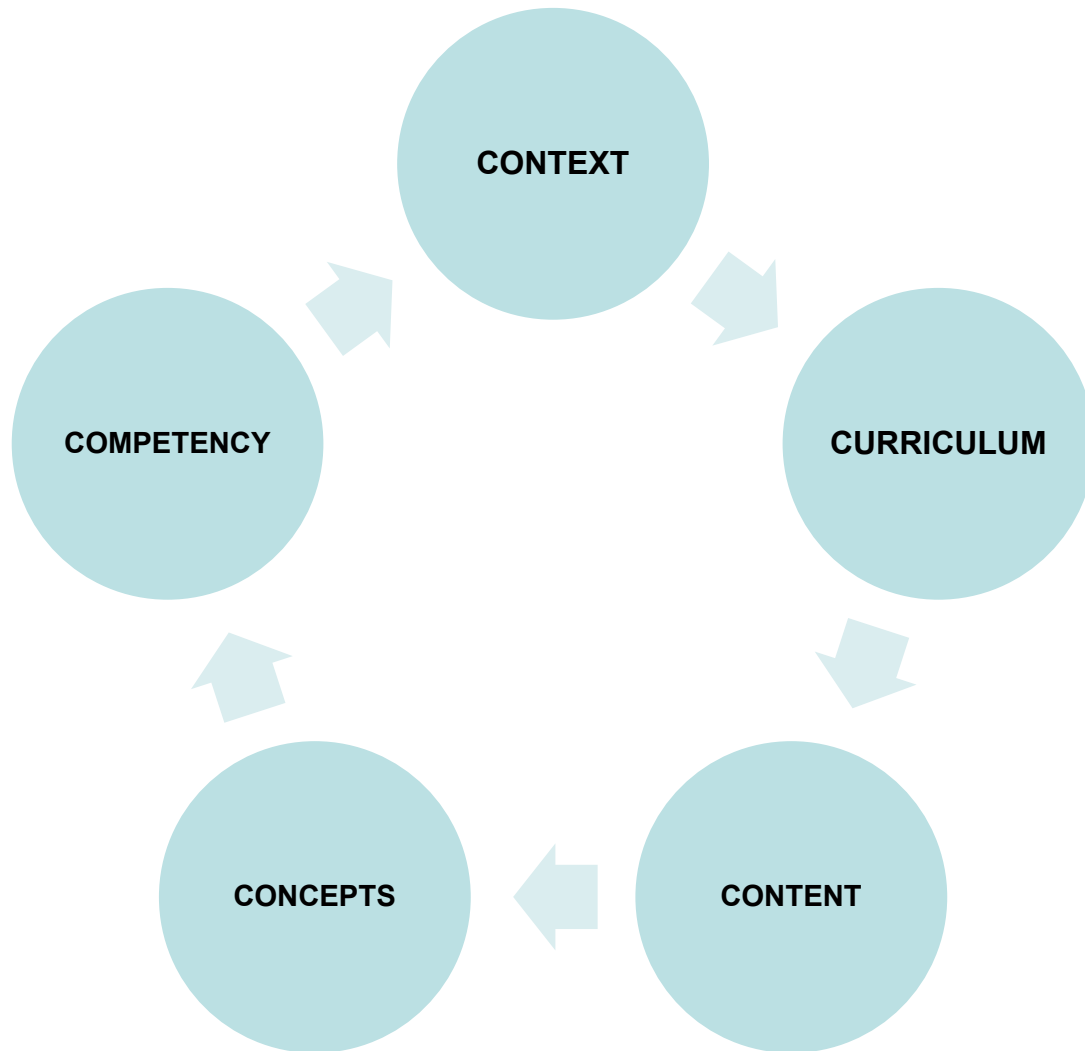


What are the 21st Century Skills?





A new philosophy: From Context to Competency





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Modeling Division

$$4 \div \frac{1}{2}$$



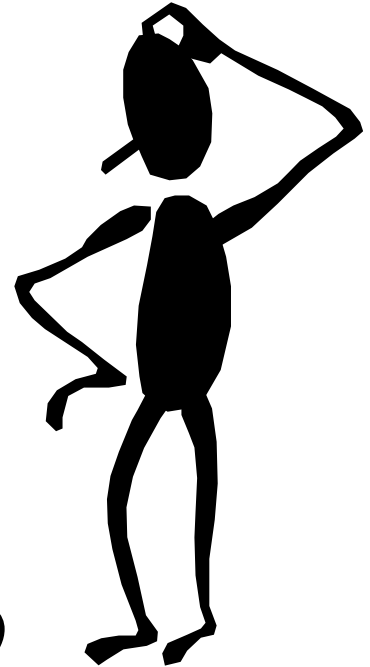
$$4$$

$$\frac{1}{2}$$

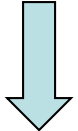
**Flip and
Multiply**



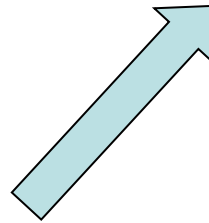
$$4 \times \frac{2}{1}$$



Modeling Division

$$4 \div \frac{1}{2}$$

$$\frac{4}{\frac{1}{2}}$$

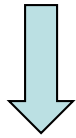
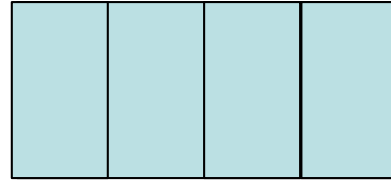
$$\frac{4}{\frac{1}{2}} \times \frac{\frac{2}{1}}{\frac{2}{1}} = 4 \times \frac{2}{1} = 8$$



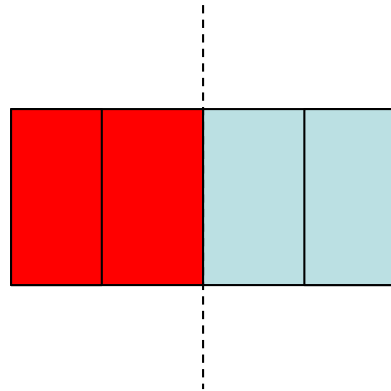
**Multiply and
Divide by it
reciprocal**

Modeling Division – Another Try

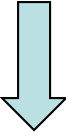
$$4 \div 2$$

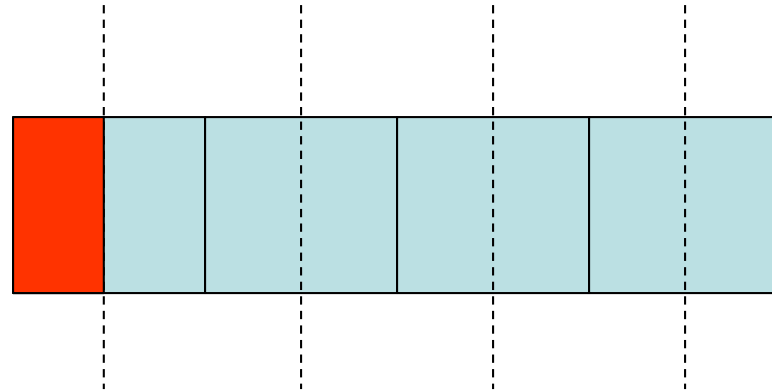


How many
twos are in
4?



Modeling Division – Another Try

$$4 \div \frac{1}{2}$$




How many
halves are
in **4**?

Understanding Learning Styles

Passagiers in de Bus

$\frac{1}{3}$ deel van de passagiers stappen

bij de eerste halte uit de bus.

$\frac{1}{2}$ deel van de rest gaan bij

tweede halte uit de bus.

Nu zijn er 5 passagiers in de bus.

Hoeveel passagiers waren er in het begin in de bus?

BE-STREAMING Videos



VISUAL
See to Learn

Geheel = $\frac{3}{3}$ deel

$\frac{1}{3}$ deel stapt uit

Over/rest $\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$ deel

$\frac{1}{2}$ deel van de rest

$\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ deel

Over is $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ deel

Over 5 passagiers = $\frac{1}{3}$ deel

Geheel $\frac{1}{3}$ deel dus $3 \times 5 = 15$



$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

5 5 5

$3 \times 5 = 15$



KINESTHETIC
Move to Learn

AUDITORY
Hear to Learn



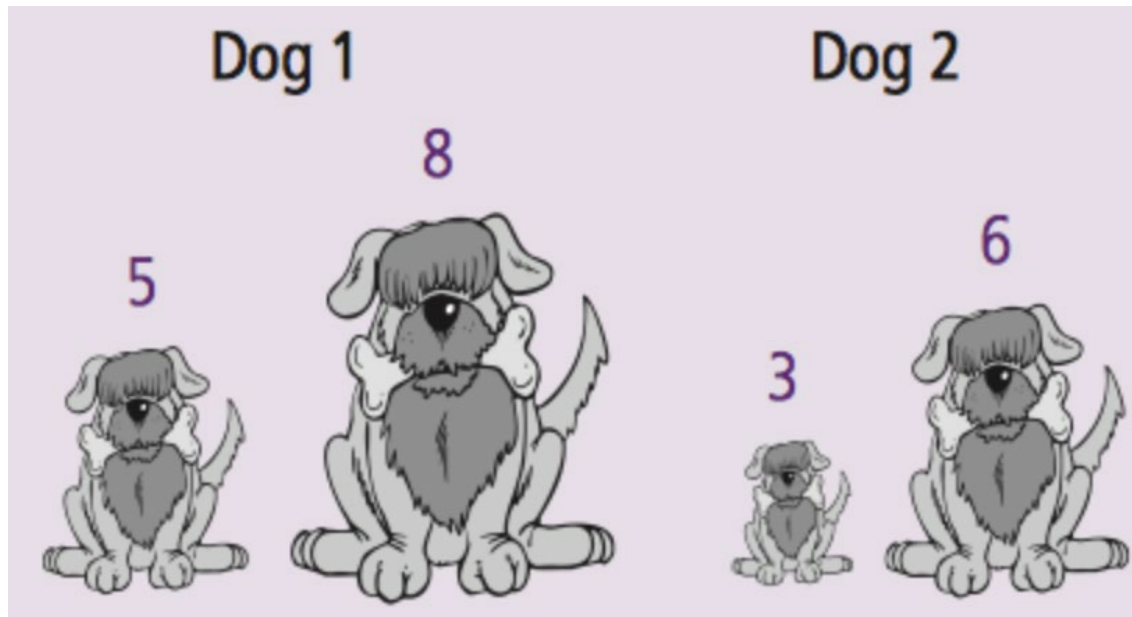
geheel

$\frac{3}{3}$ deel



Which dog grew more?

- If one dog grows from **5** kg to **8** kg and another dog grows from **3** kg to **6** kg, which dog grew more?



Which is a better investment?

Pat: Invest \$300 → Receive \$600

Aaron: Invest \$500 → Receive \$800



Why?



Why the numbers matter?

RELATIVE RISK

EATING PROCESSED MEAT
INCREASES THE RISK OF
COLON CANCER 18%
($5.9/5.0 = 1.18$)
RELATIVE RISK FOR
SMOKING & CANCER IS 2300%

ABSOLUTE RISK

EATING PROCESSED MEAT
INCREASED ASSOCIATED
COLON CANCER INCIDENCE
FROM 5 PER 100
TO 5.9 PER 100
(0.9% INCREASE)

RELATIVE RISK

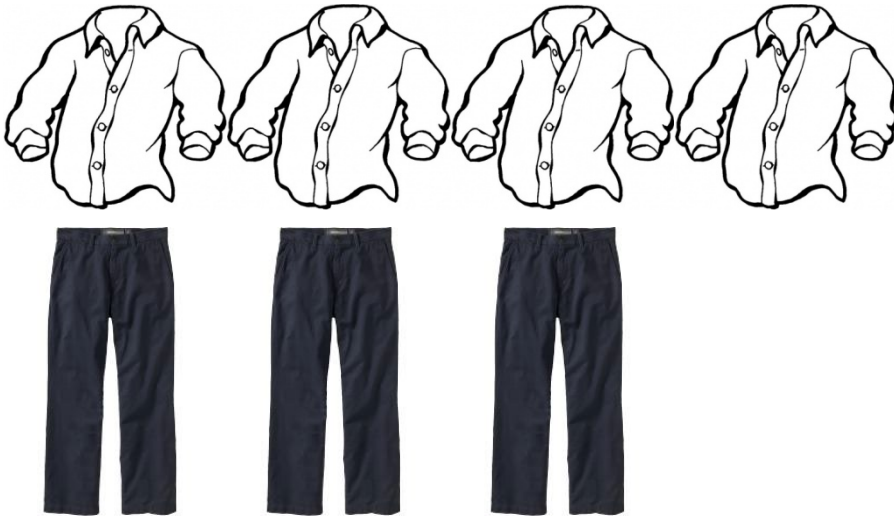
"New wonder drug
reduces heart
attack risk 50%"

ABSOLUTE RISK

"New wonder drug
reduced heart attacks
from from 2 per 100
to 1 per 100"

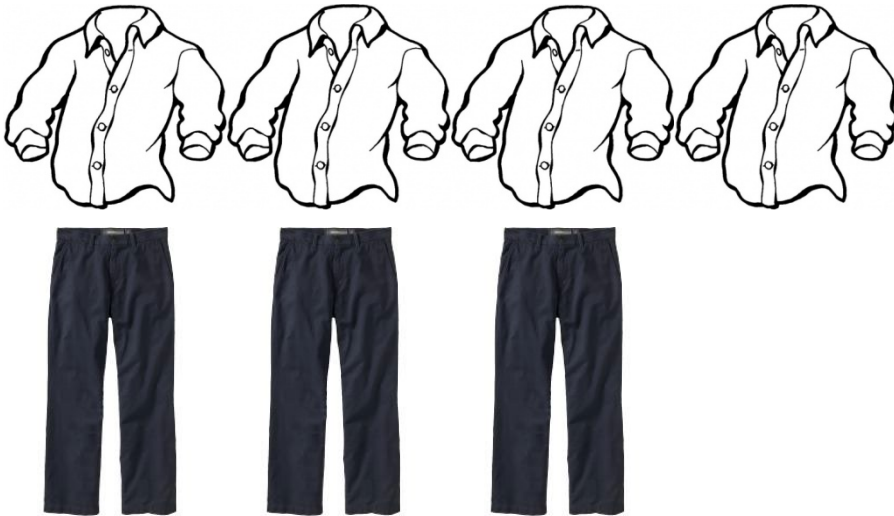
Finding Dress Cost

- Four shirts and three pants cost \$37. Three shirts and four pants cost \$33. Find the cost of one shirt.



Finding Dress Cost

- **Four** shirts and **three** pants cost **\$37**. **Three** shirts and **four** pants cost **\$33**. Find the cost of one shirt.

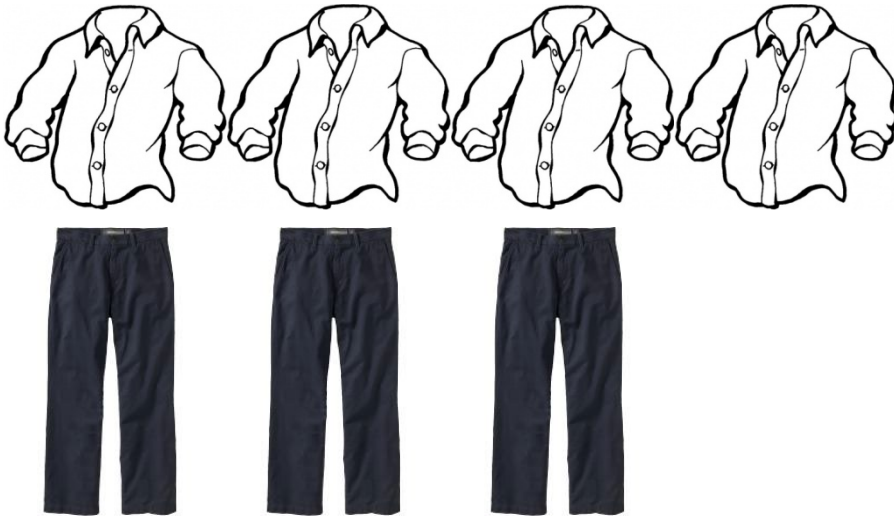


$$4S + 3P = 37$$

$$3S + 4P = 33$$

Finding Dress Cost

- **Four** shirts and **three** pants cost **\$37**. **Three** shirts and **four** pants cost **\$33**. Find the cost of one shirt.



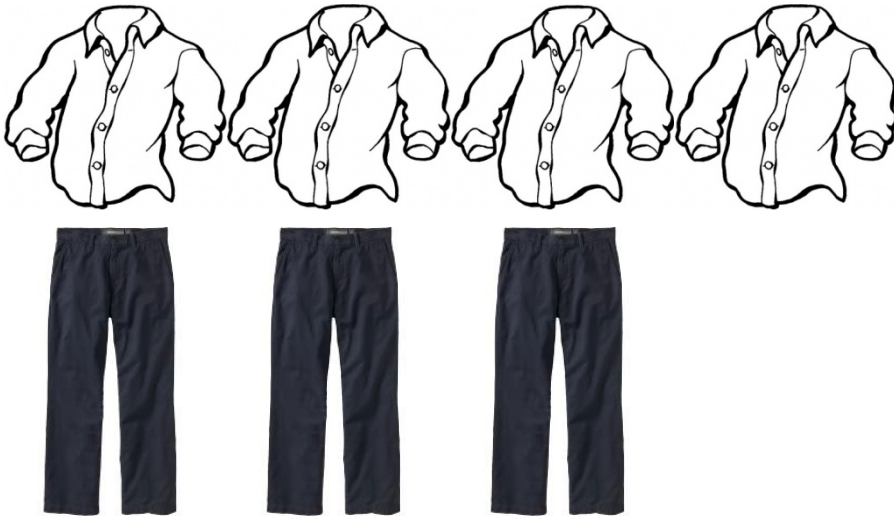
$$4S + 3P = 37$$

$$3S + 4P = 33$$

- Substitution
- Linear Addition
- Graphing
- Matrix Inverse
- Cramer's Rule
- Gaussian Elimination
- More...

Modeling dress cost

- Four shirts and three pants cost \$37. Three shirts and four pants cost \$33. Find the cost of one shirt.



$$4S + 3P = 37$$

$$5S + 2P = 41$$

$$6S + P = 45$$

$$7S = 49$$

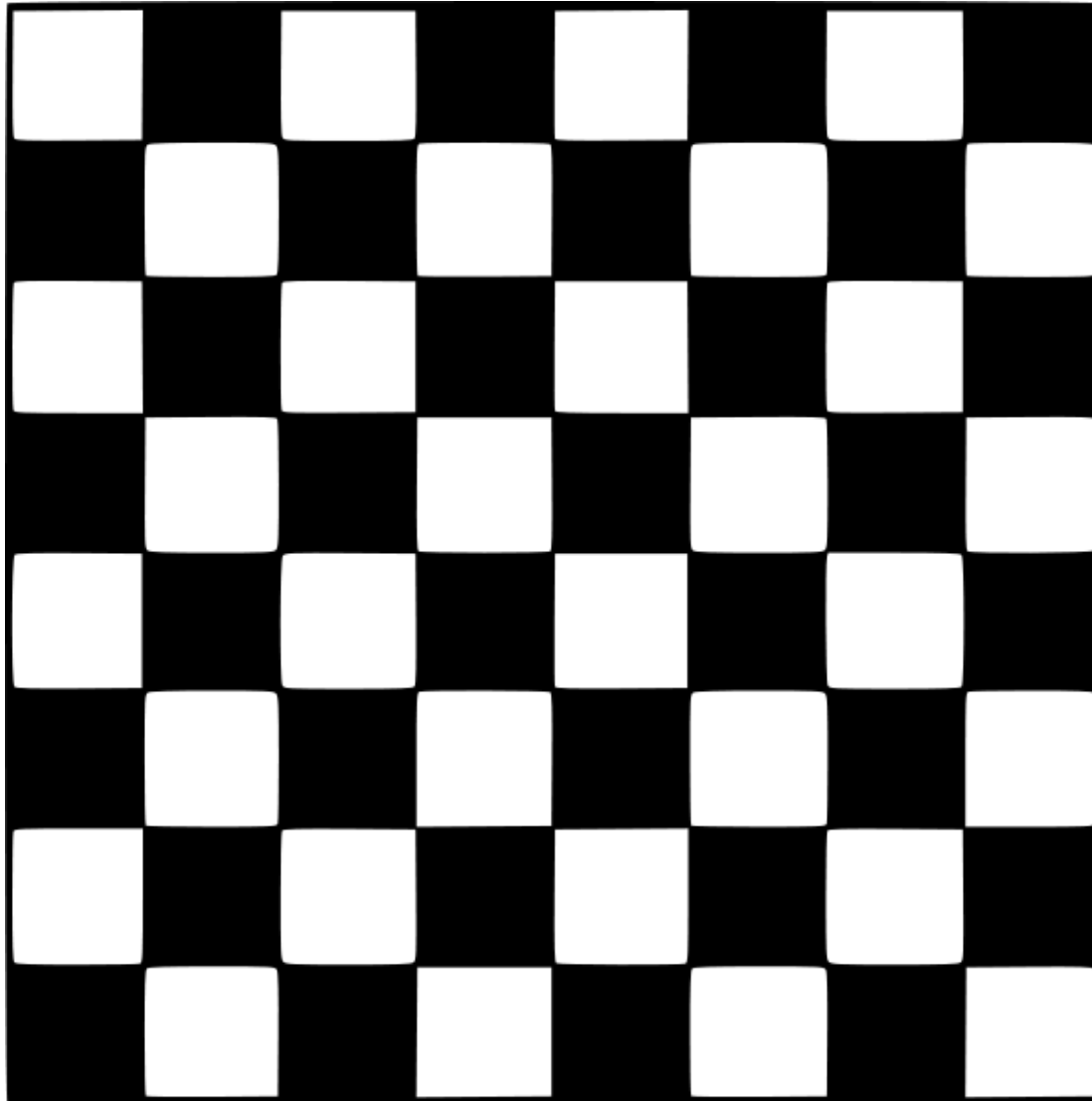
$$S = 7$$

$$4S + 3P = 37$$

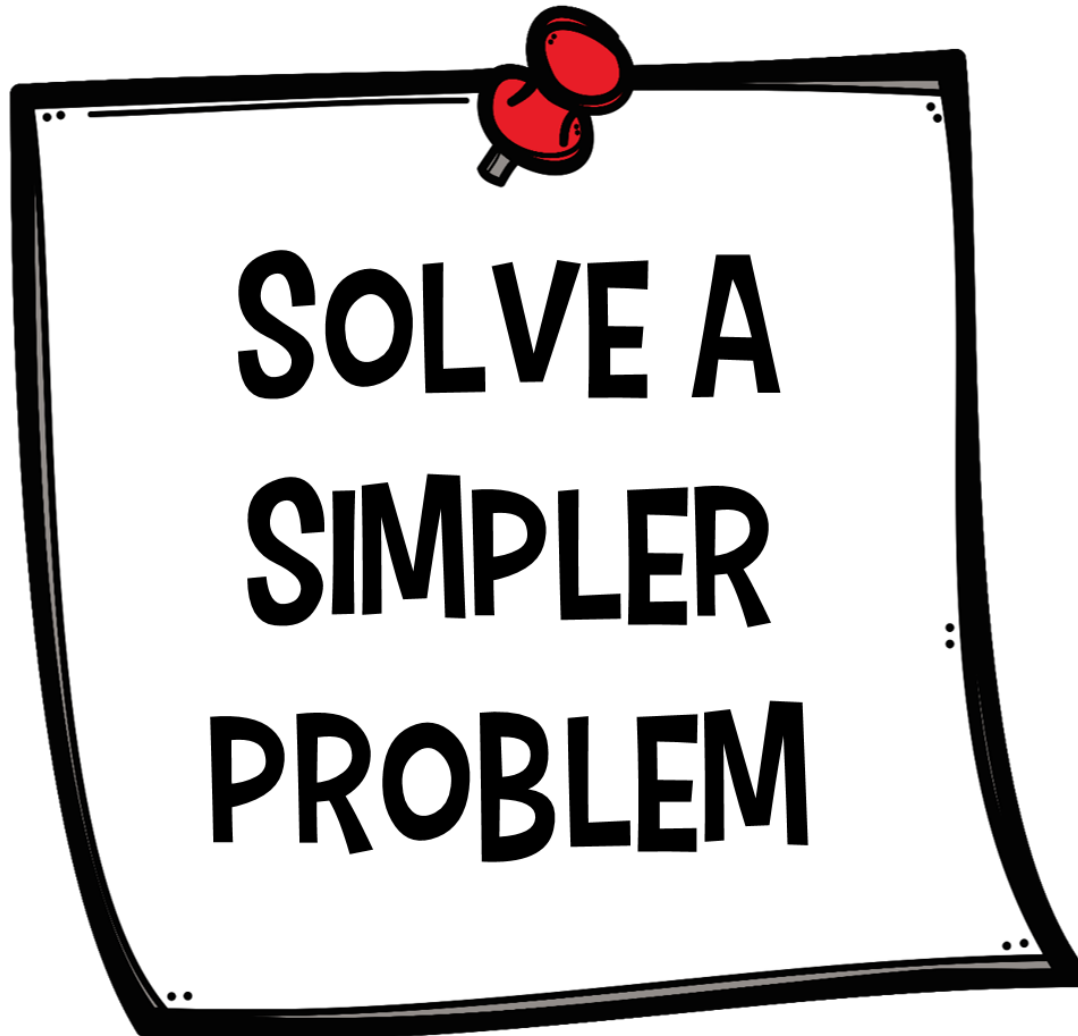
$$3S + 4P = 33$$

$$S - P = 4$$

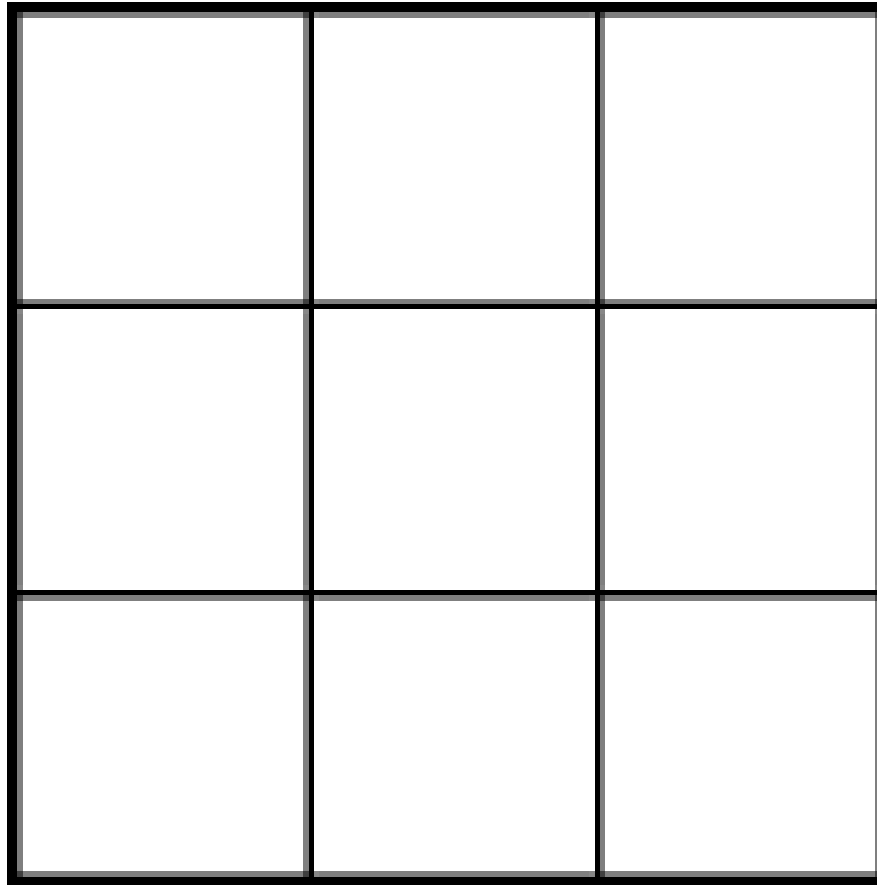
How many **squares** do you see?



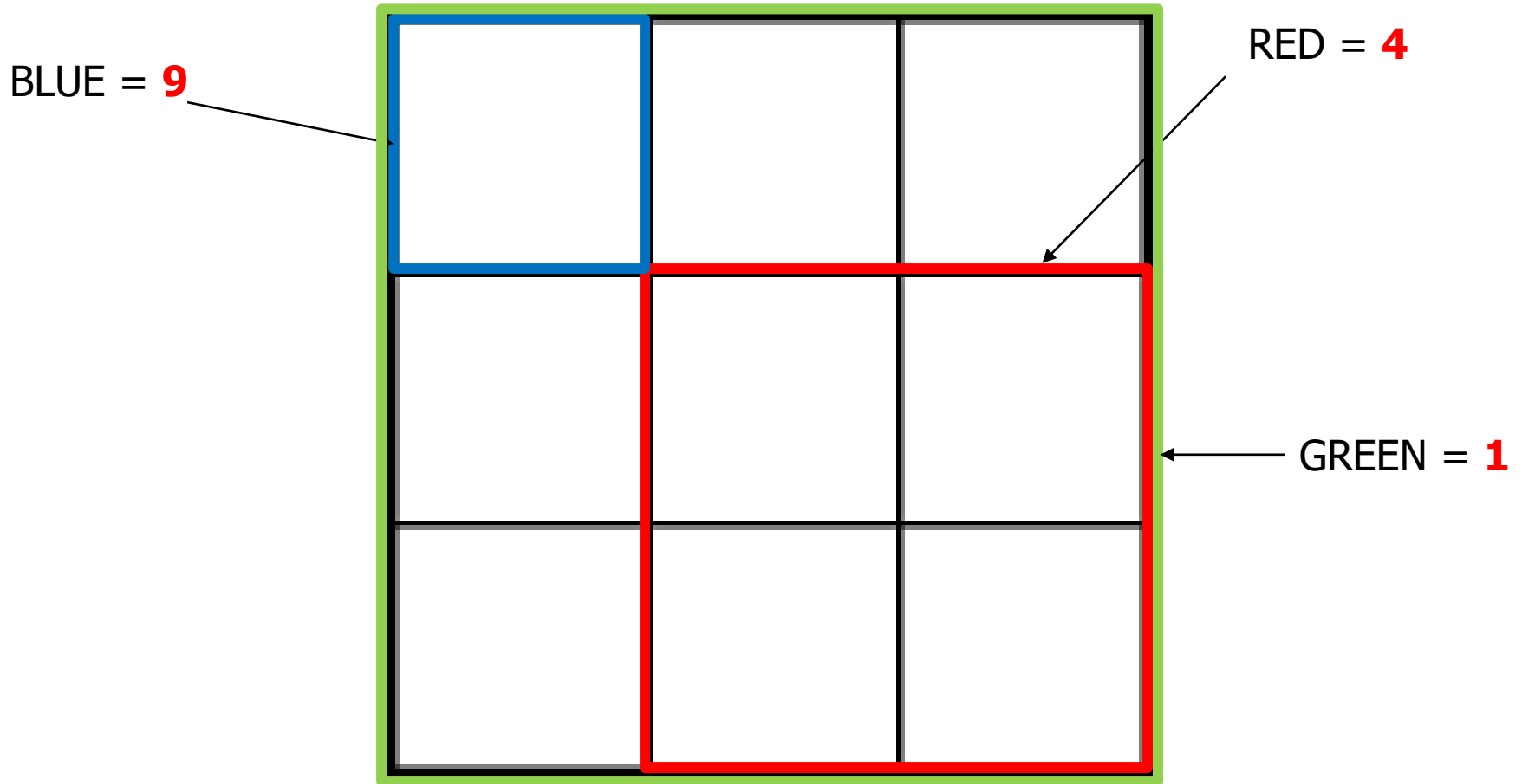
Teaching Moment



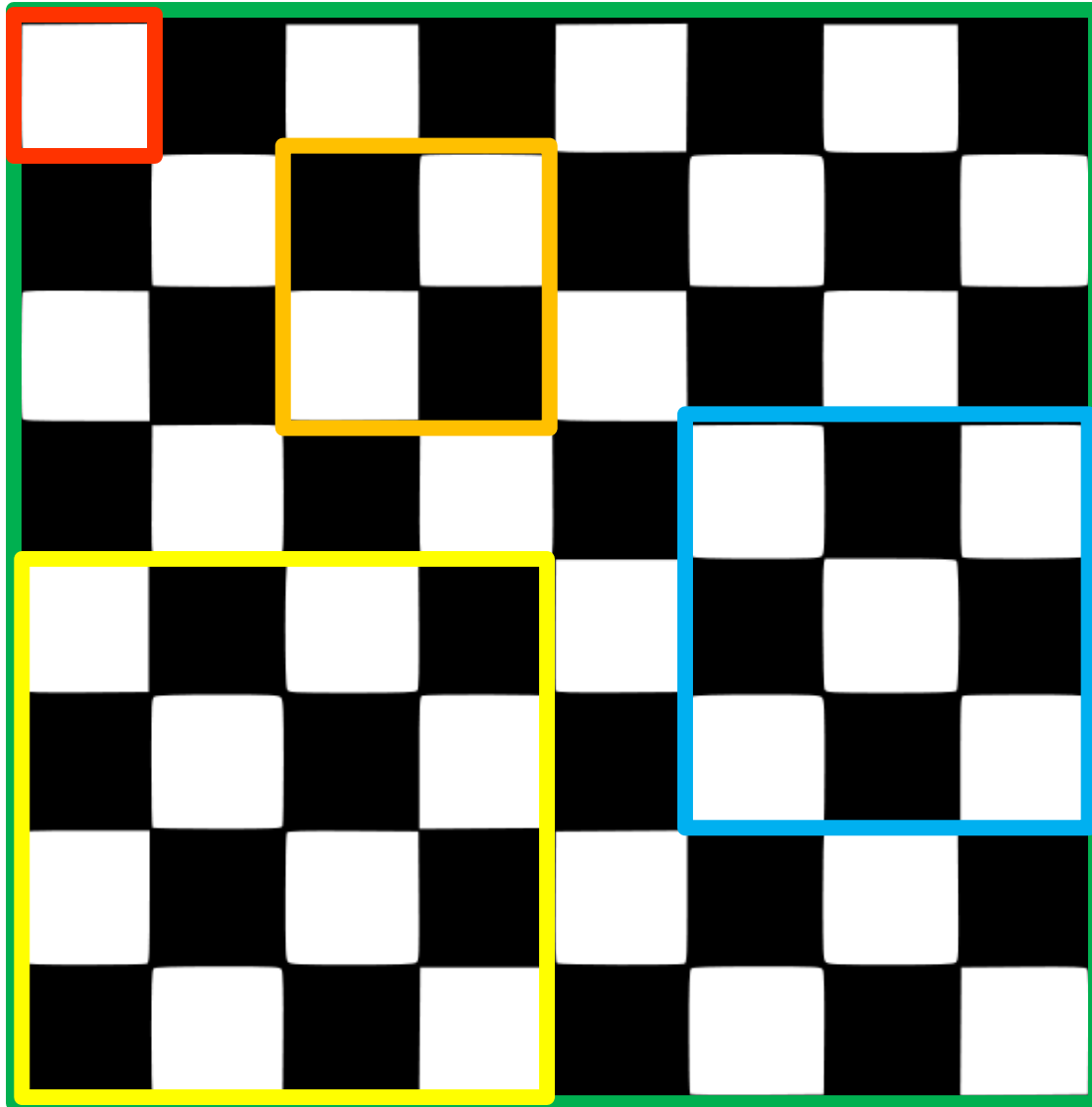
How many **squares** do you see?



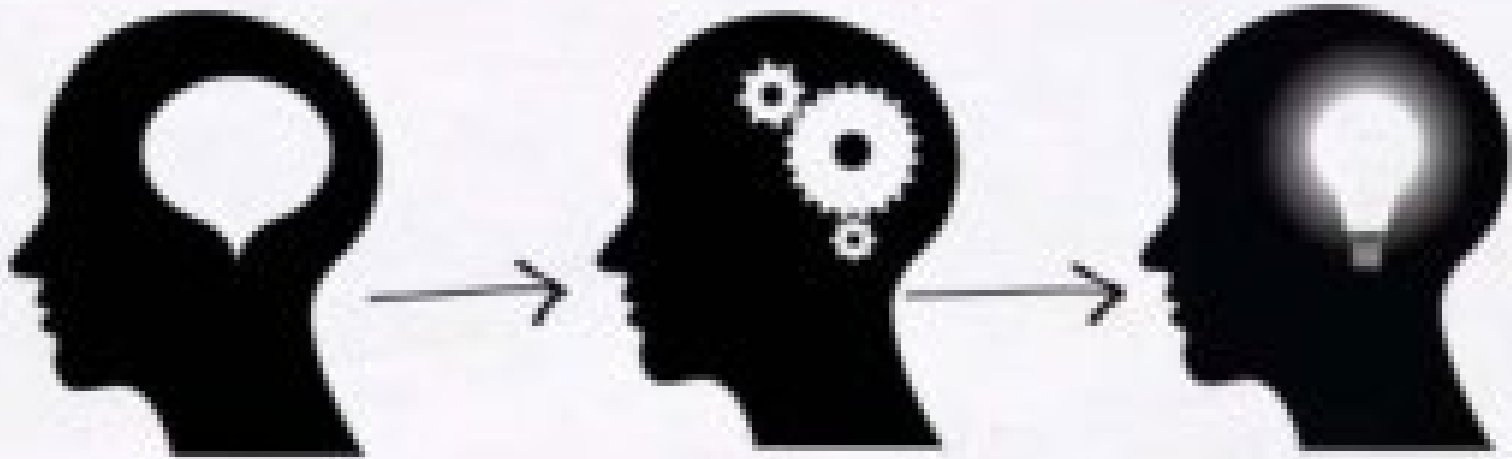
How many **squares** do you see?



How many **squares** do you see?



Critical Thinking





$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$



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SINGAPORE

Teach less, Learn More

- **Teacher-directed**

- Direct Instruction
- Knowledge
- Content
- Basic Skills
- Facts and Principles
- Theory
- Curriculum
- Time-slotted
- One-size-fits-all
- Competitive
- Classroom
- Text-based
- Summative Tests
- Learning for School

- **Learner-Centered**

- Interactive exchange
- Skills
- Process
- Applied Skills
- Questions and Problems
- Practice
- Projects
- On-demand
- Personalized
- Collaborative
- Global Community
- Web-based
- Formative evaluations
- Learning for Life

FINLAND

Learning by Doing

KIDS HAVE MORE TIME TO BE KIDS.

AN AVERAGE US 5TH GRADER HAS 50 MIN. OF HOMEWORK PER DAY.



Finnish students rarely do homework until their teens.

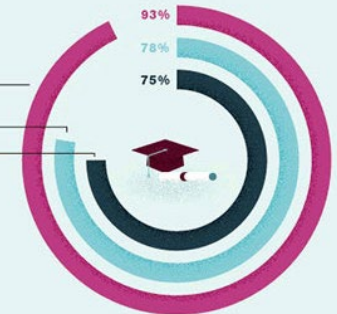


And while US elementary students average 27 minutes of recess...

... STUDENTS IN FINLAND GET ABOUT 75 MINUTES A DAY.

Finland's school system accomplishes some impressive feats:

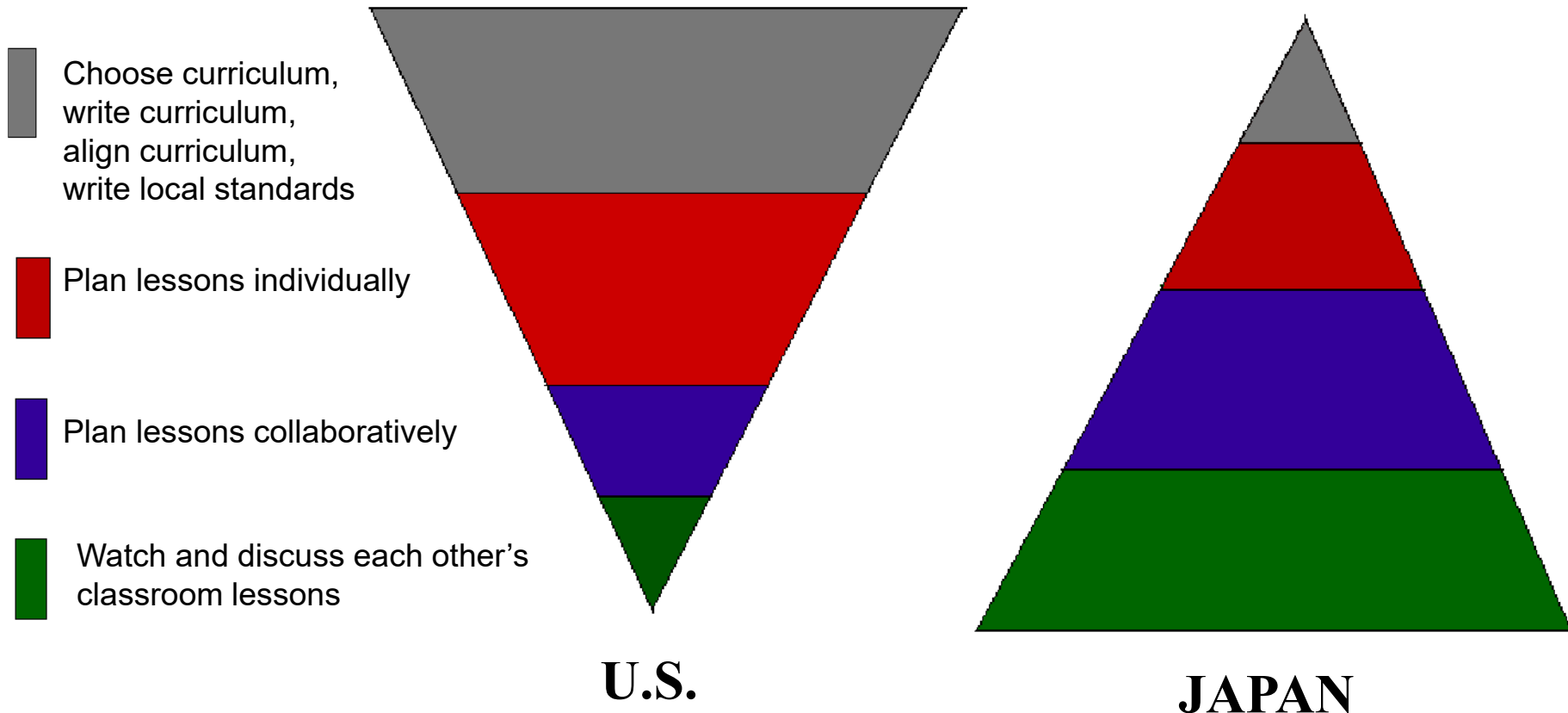
THEIR HIGH SCHOOL GRADUATION RATE IS AT 93%.
COMPARED TO 78% IN CANADA.
AND 75% IN THE US.



That's the highest rate in all of Europe.

JAPAN

Teachers' Activities to Improve Instruction



Experiential Learning



Innovative Learning

QUIZ!



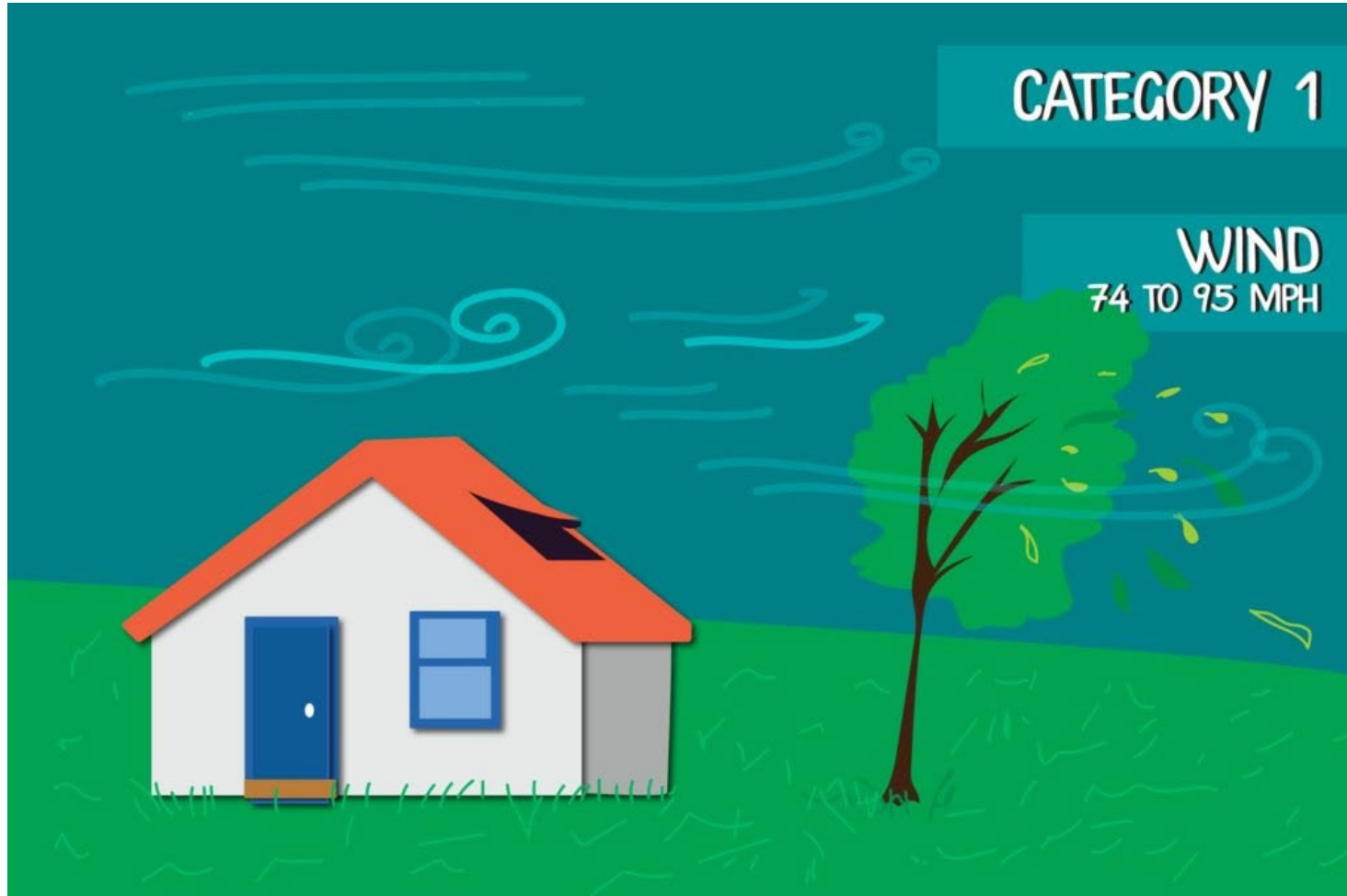
Innovative Learning



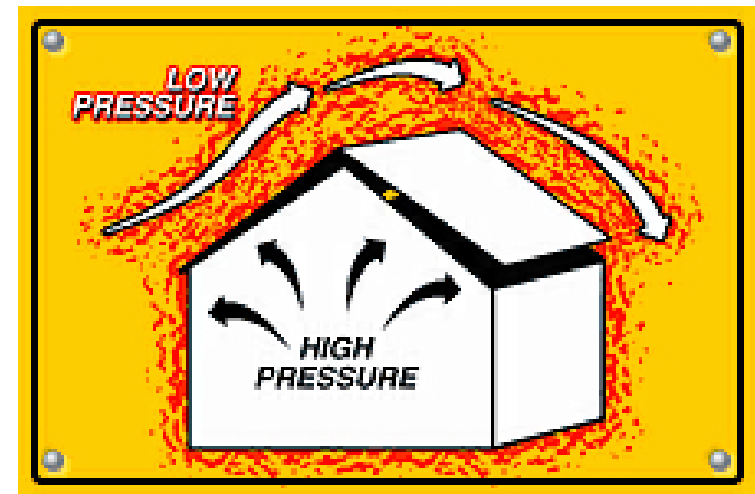
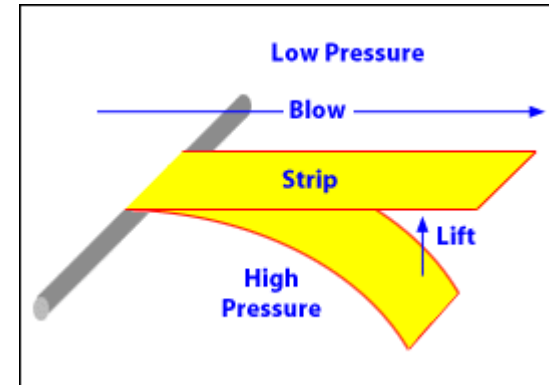
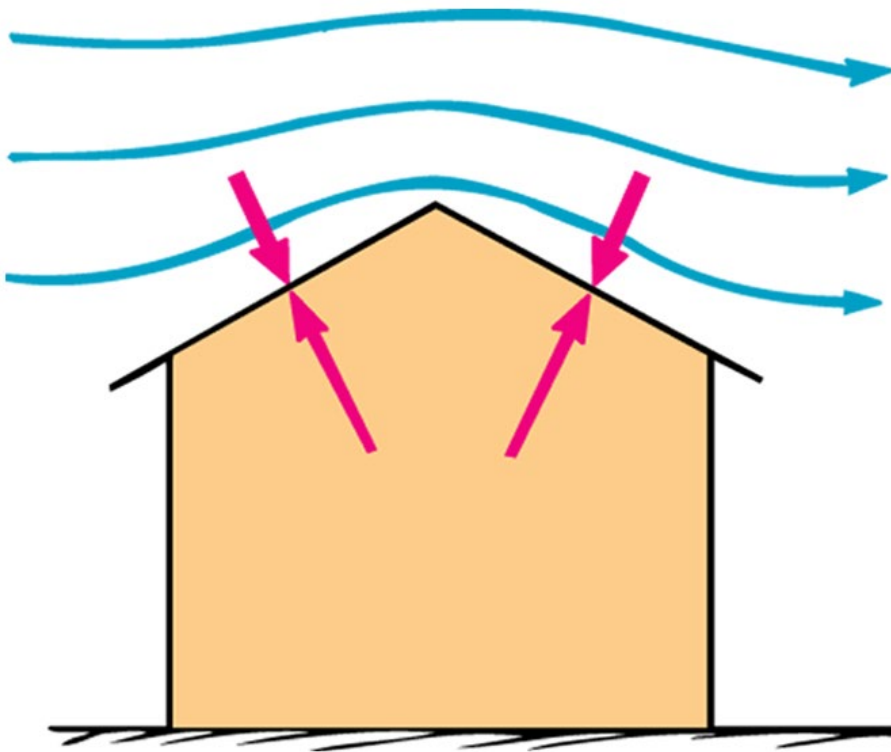
Inquiry-based Learning



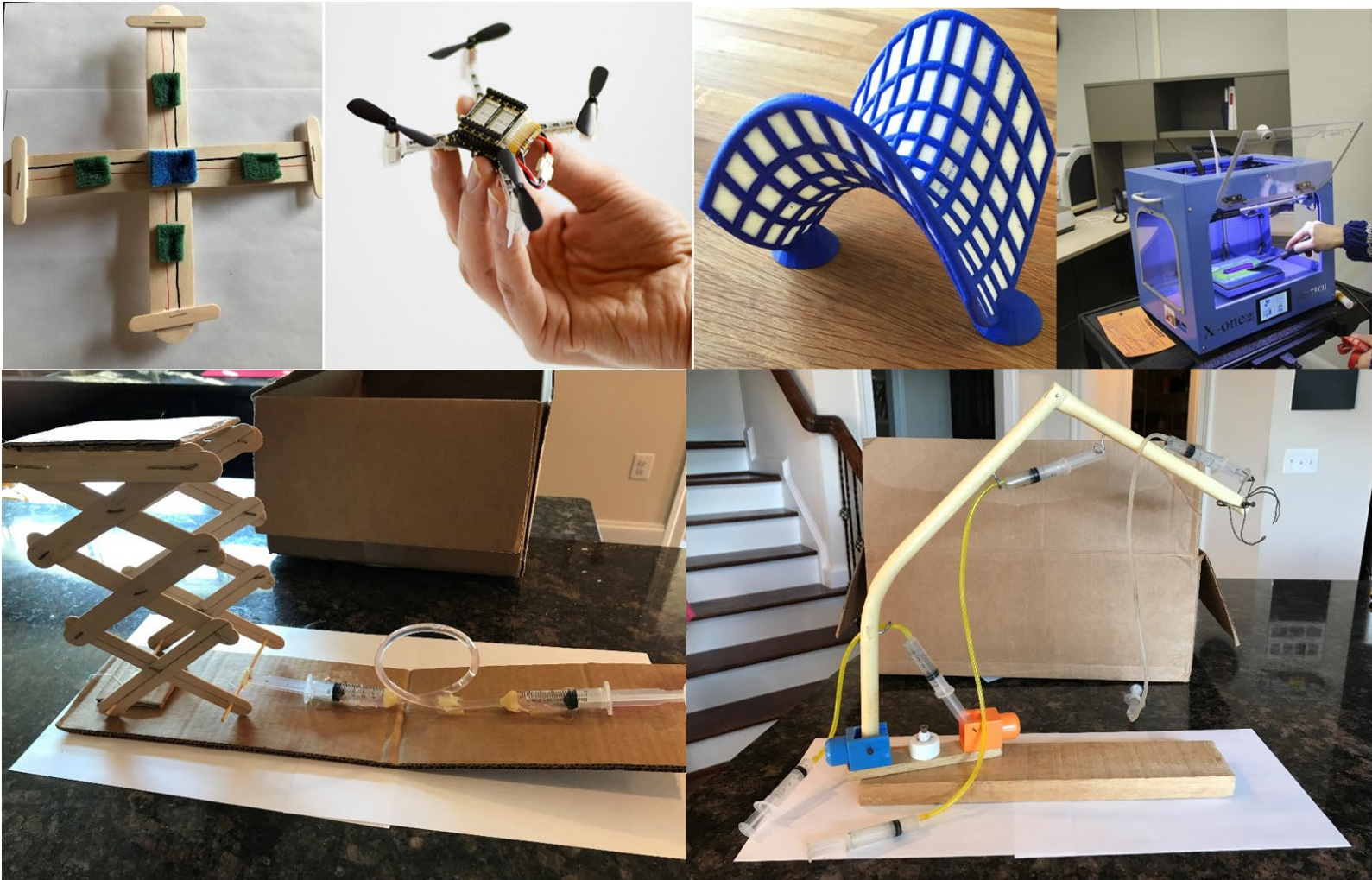
Why do roofs fly?



Bernoulli's Principle



Learning by Doing



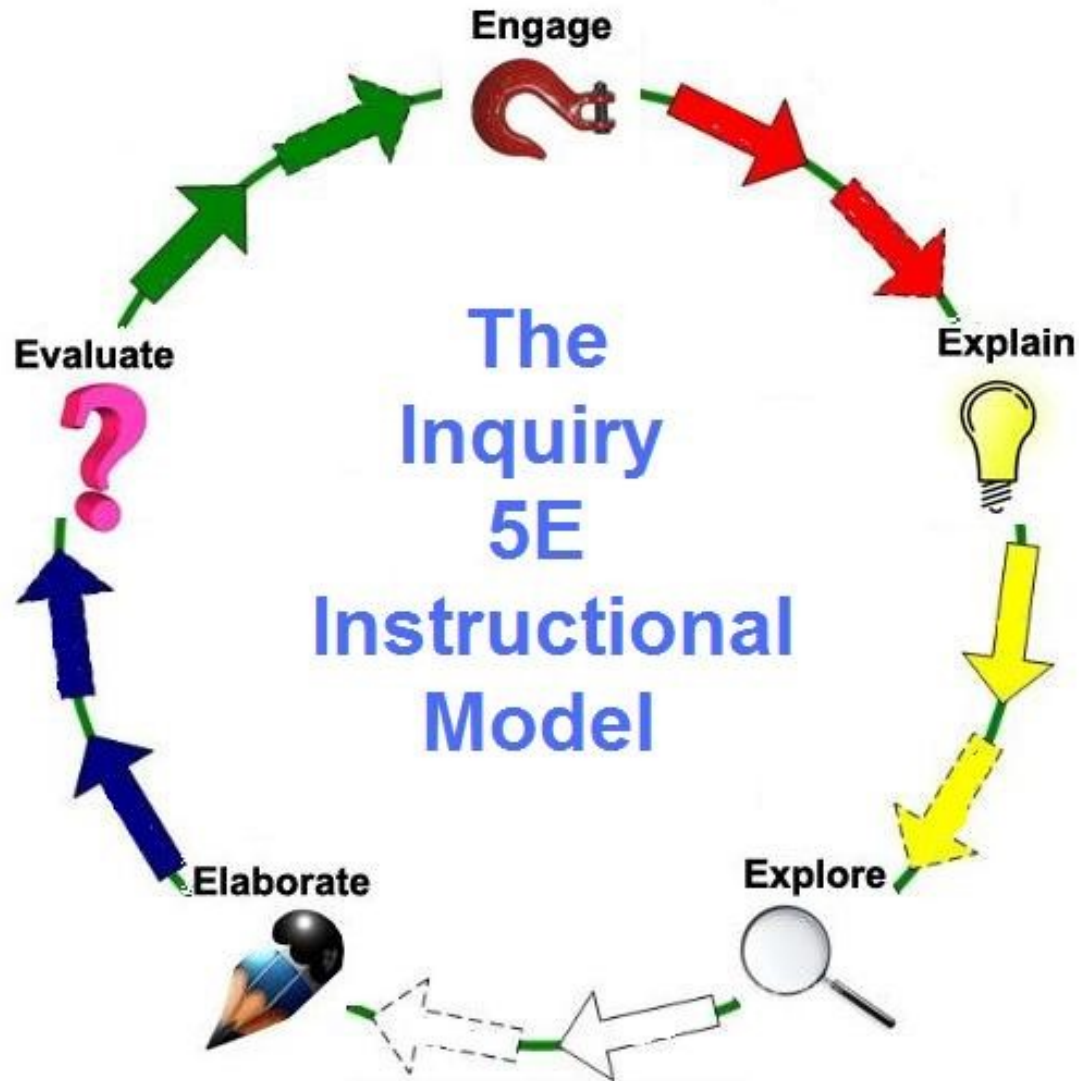
Collaborative Learning



Active Learning



Integrated Teaching



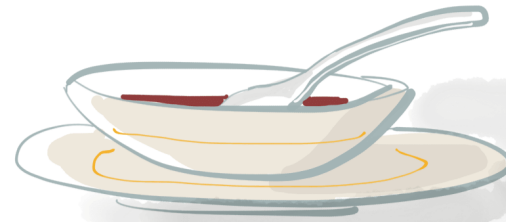
Performance based Assessment

Avaliação

FORMATIVE SUMMATIVE



WHEN THE CHEF
TASTES THE SOUP

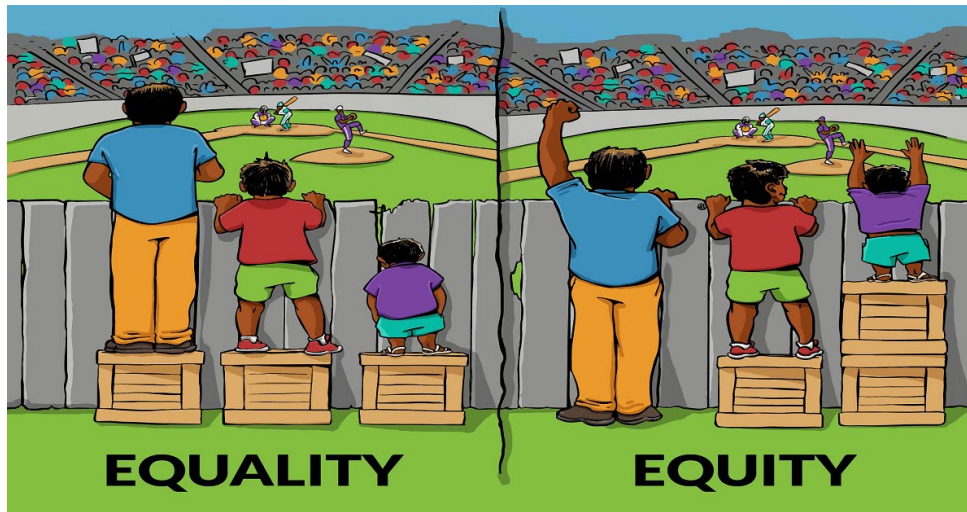
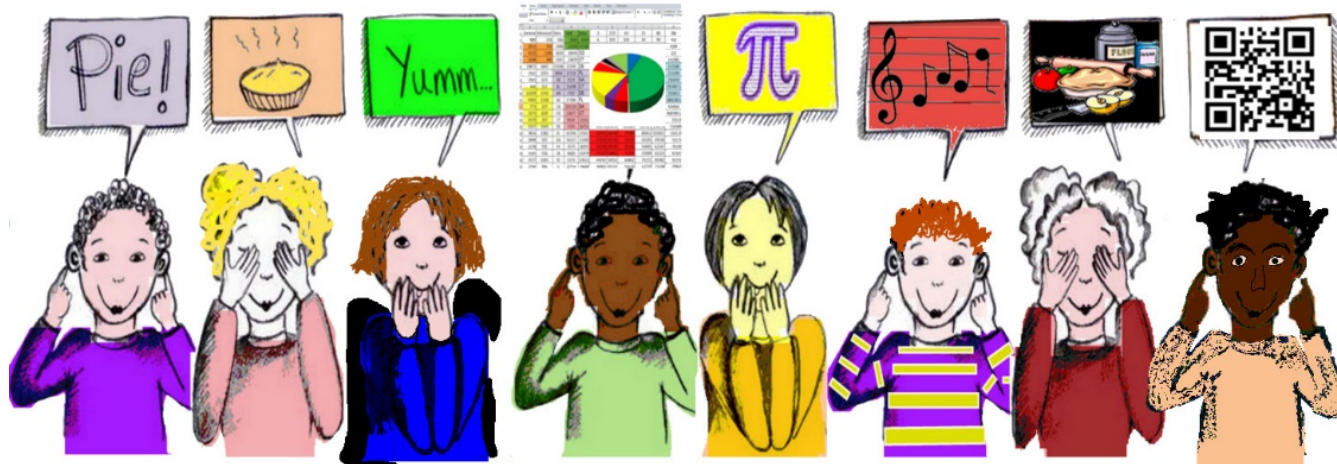


WHEN THE GUESTS
TASTE THE SOUP

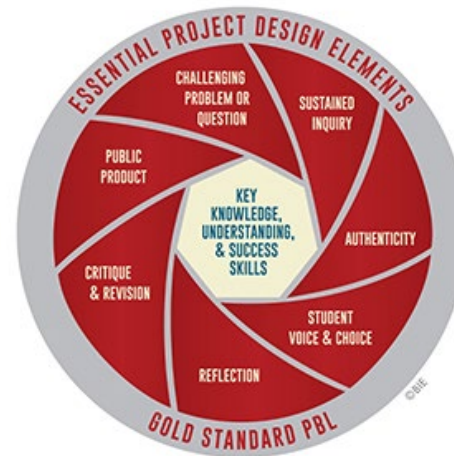
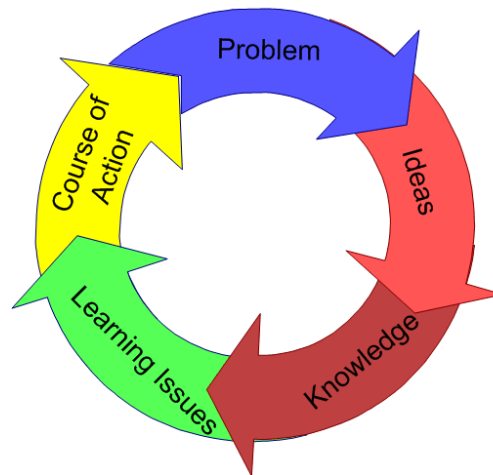
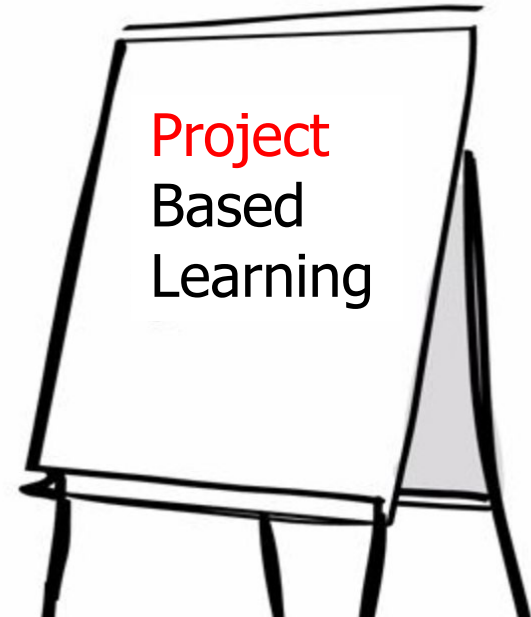
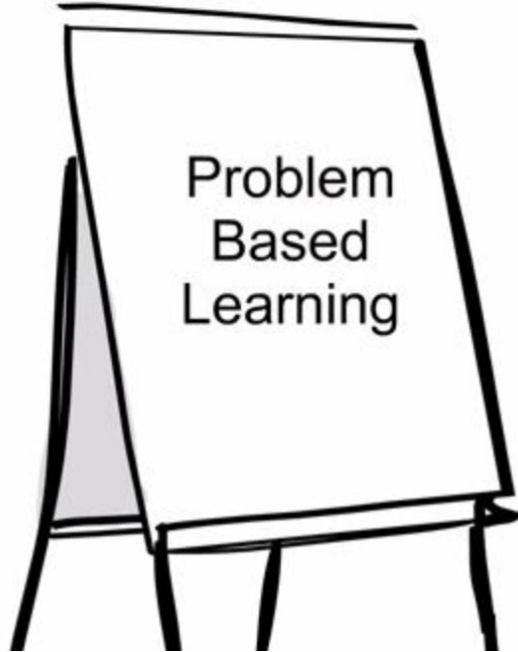


Differentiated Instruction

Aprendizado diferenciado



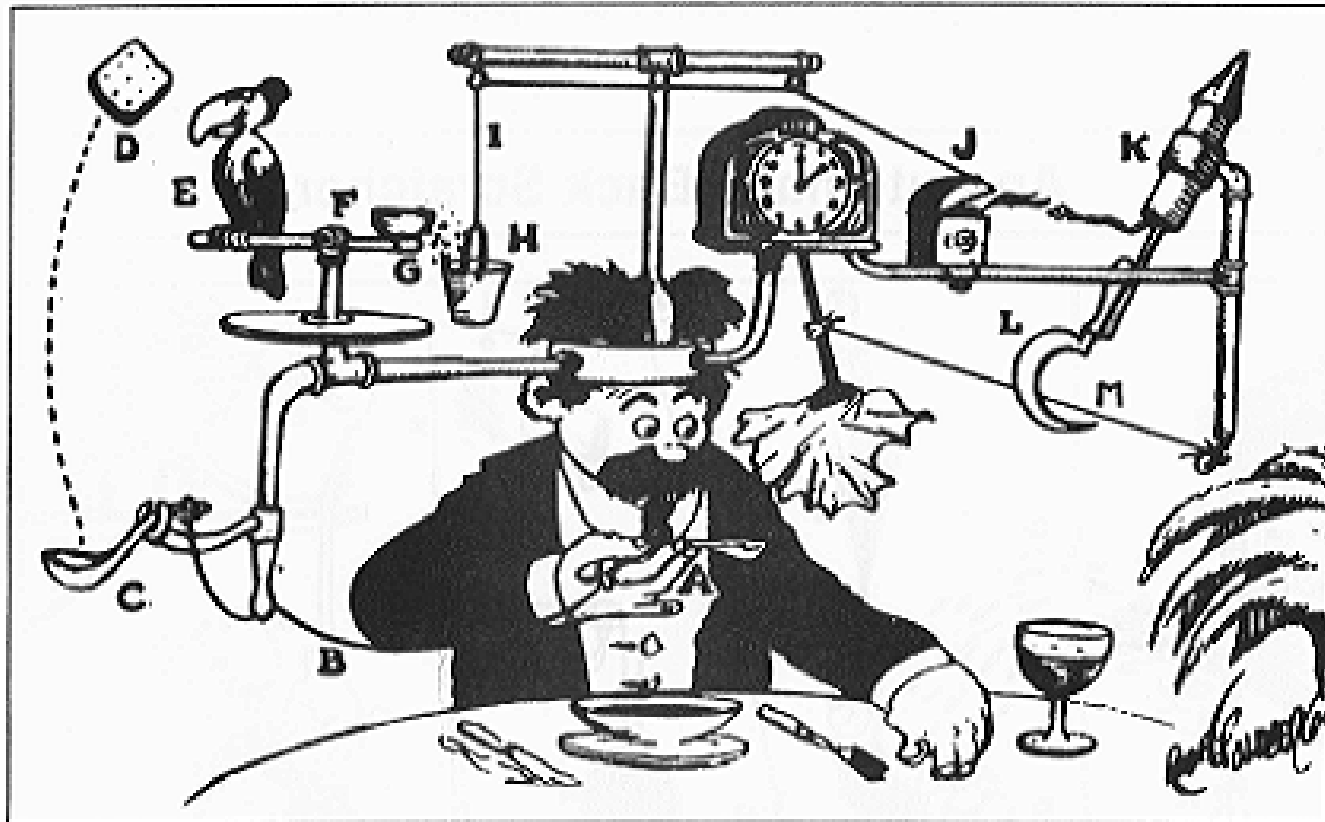
PBL



Problem Based Learning

Rube-Goldberg Project

Self-Operating Napkin



Project Based Learning

Save the Turtles



Challenge Based Learning



Make solar energy economical



Provide energy from fusion



Develop carbon sequestration methods



Manage the nitrogen cycle



Provide access to clean water



Restore and improve urban infrastructure



Advance health informatics



Engineer better medicines



Reverse-engineer the brain



Prevent nuclear terror



Secure cyberspace



Enhance virtual reality



Advance personalized learning



Engineer the tools of scientific discovery

Challenge Based Learning



 **SUSTAINABLE DEVELOPMENT GOALS**



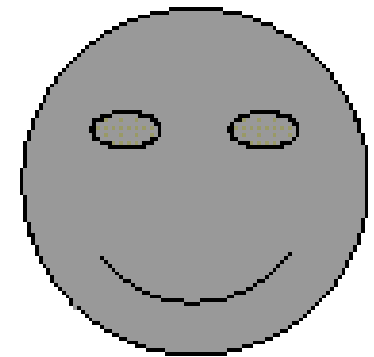
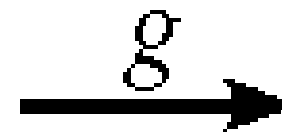
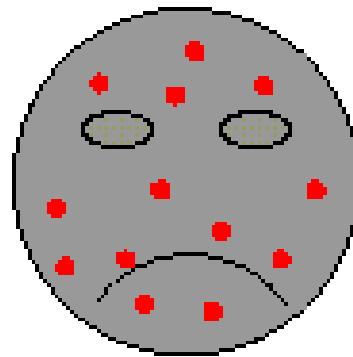
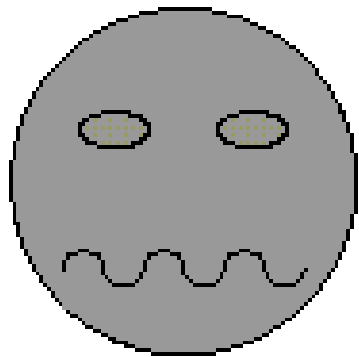
Disease Dynamics Modeling
Identifying the threshold or tipping point
(Kermack and McKendrick, 1927, 1932, 1933)

- Individuals are found in three stages

– **Susceptible**

– **Infected**

– **Recovered**



Susceptible

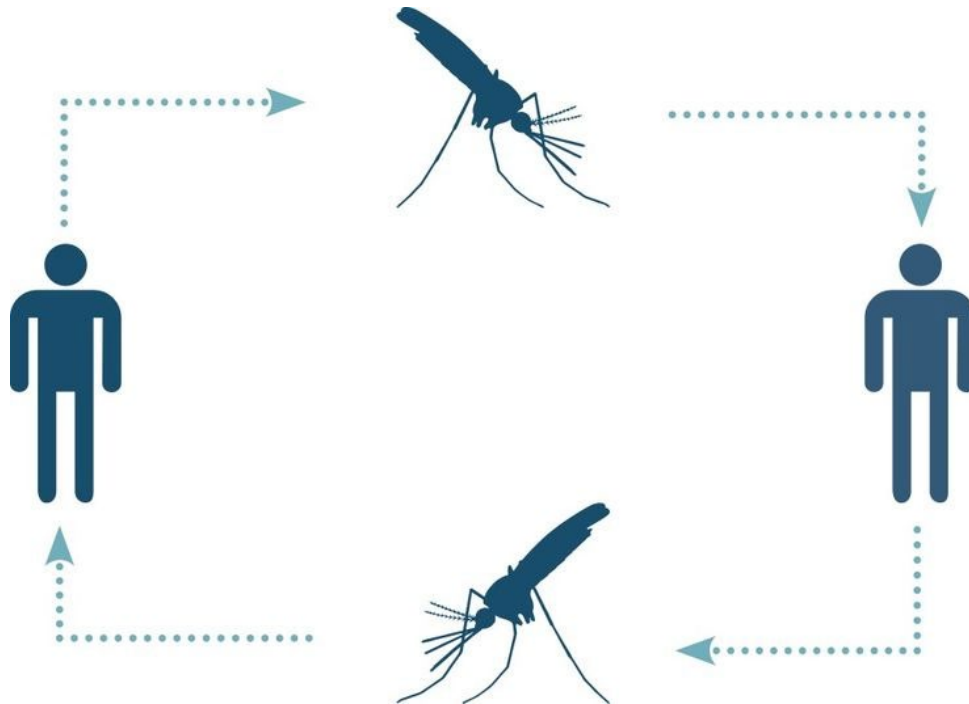
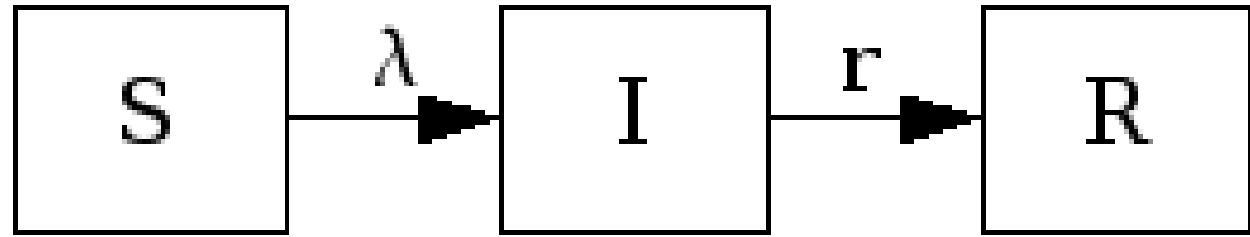
Infectious

Recovered

Mathematics of Spread of Disease



SIR



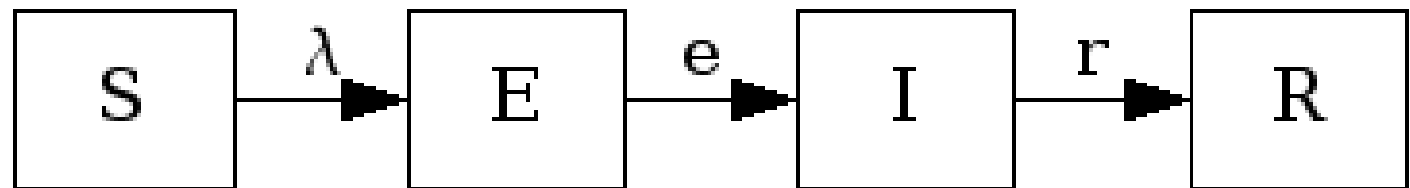
$$\frac{dS}{dt} = -\lambda S I$$

$$\frac{dI}{dt} = \lambda S I - r I$$

$$\frac{dR}{dt} = r I$$

$$S(t) + I(t) + R(t) = N$$

SEIR



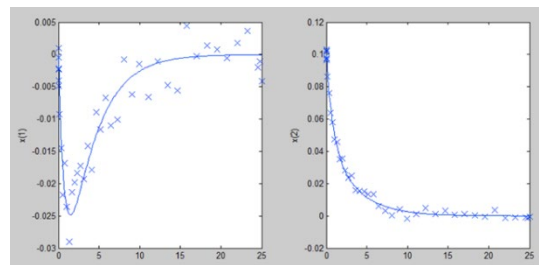
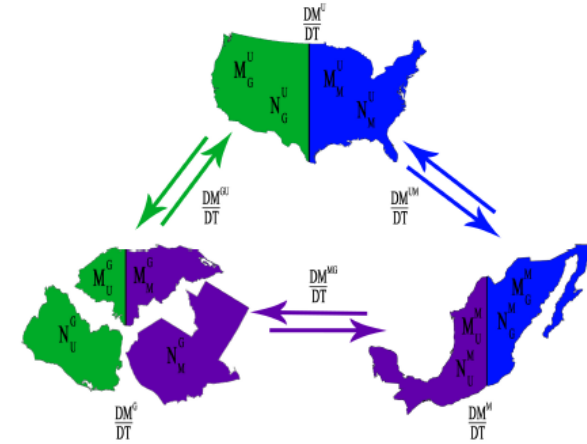
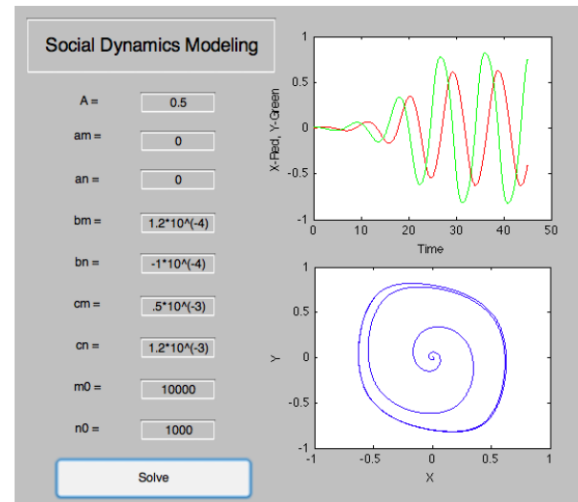
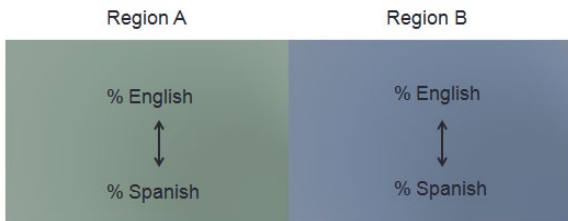
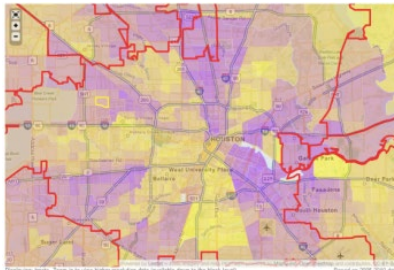
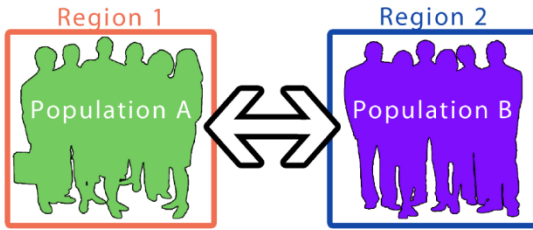
 **SUSTAINABLE DEVELOPMENT GOALS**



Global Problem Solving

- Location: Houston, TX
- Problem: Social Dynamics

$$\frac{dm_i^\alpha(t)}{dt} = \sum_{j=i, j \neq i}^s m_j^\alpha(t)w_{ij}^\alpha(t) - m_i^\alpha(t)w_{ji}^\alpha(t) + P_i$$



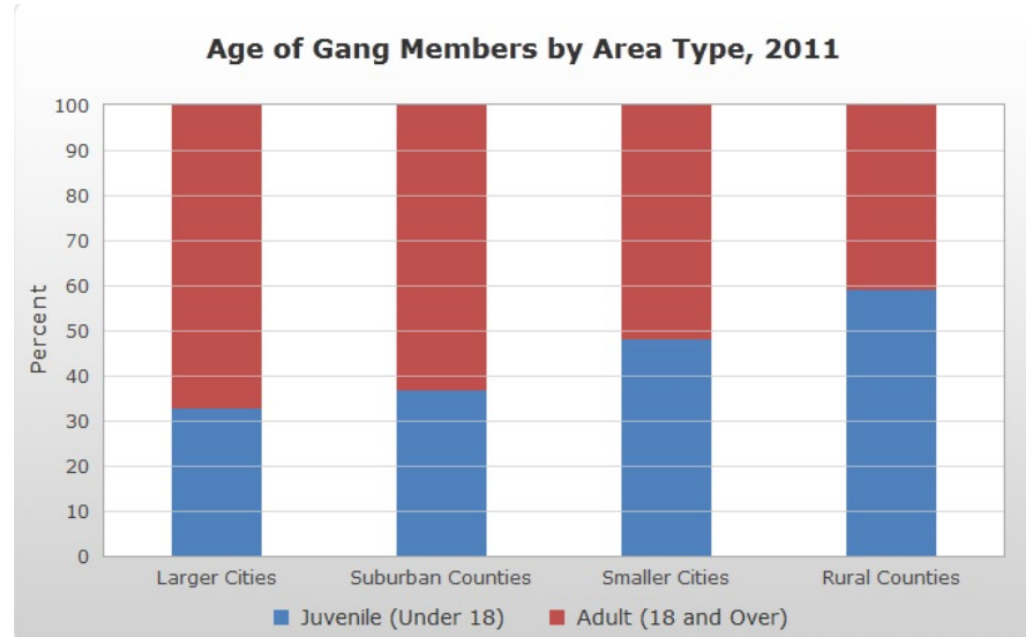
Parameter Estimation



Posters on the Hill

Global Problem Solving

- Location: Houston, TX
- Problem: Gang Recruitment

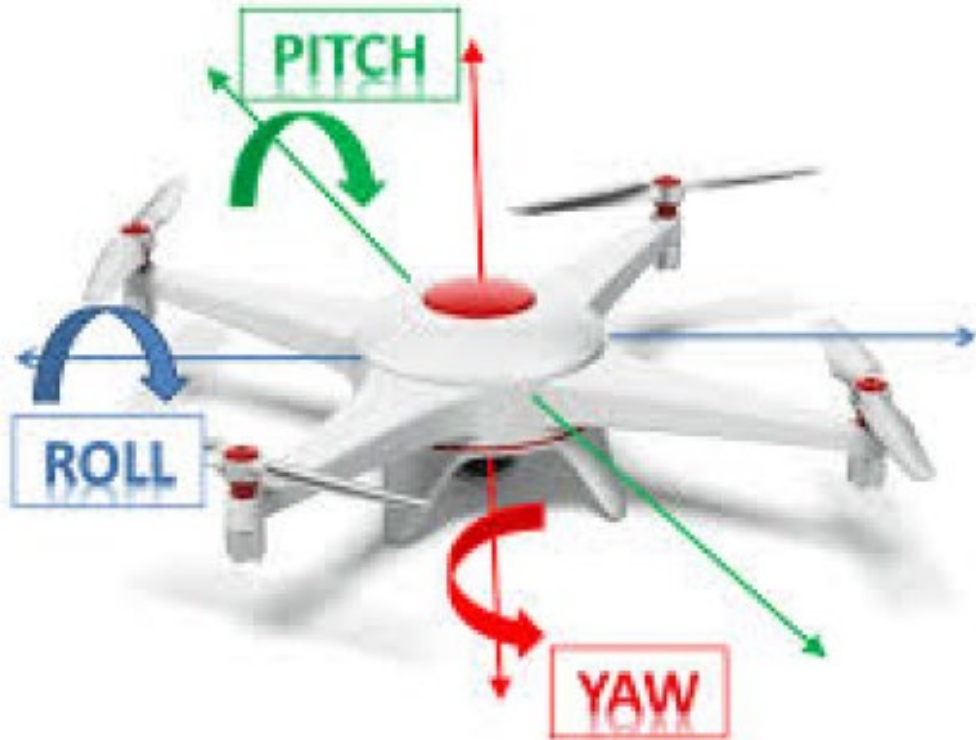


Global Problem Solving

- Location: Tanzania
- Problem: Poaching





Drones and Mathematics



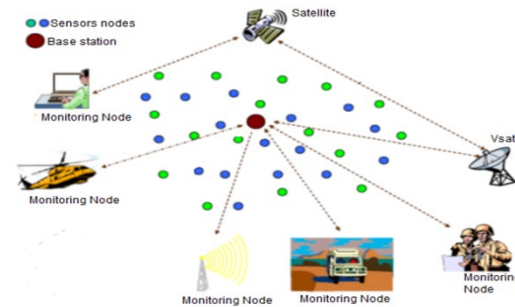
credit: norunway.com



a_1	a_2 
a_3 	a_4

Why is this a rich STEM Problem?

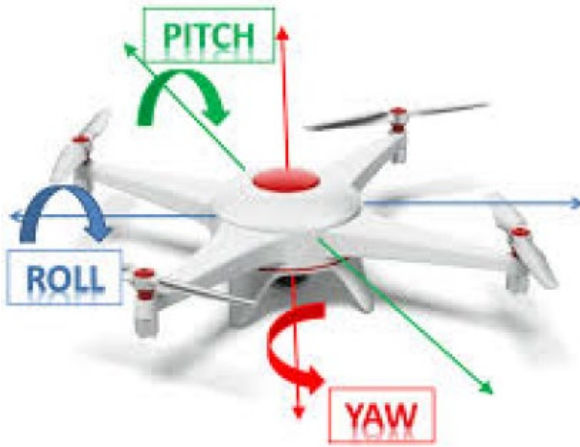
- Dynamics and Mechanics
- Search Algorithms
- Sensors and Electronics
- Control Systems and Feedback
- Communications
- Swarming
- Mapping Algorithms
- Machine Learning
- **Applications:** Anti-poaching, Remote Sensing, Agriculture, Transporting materials, Oil-gas-mineral exploration, Search & Rescue, Surveillance and many more!



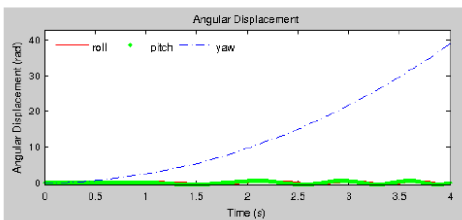
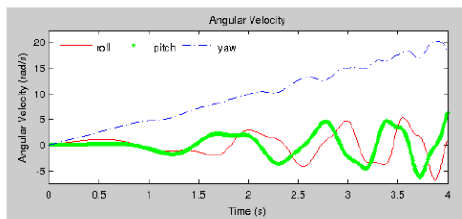
Studying Drone Dynamics using
Design Thinking and Computational Mathematics

<https://sinews.siam.org/Details-Page/applying-design-thinking-to-mathematics-research-2>

Drones and Mechanics

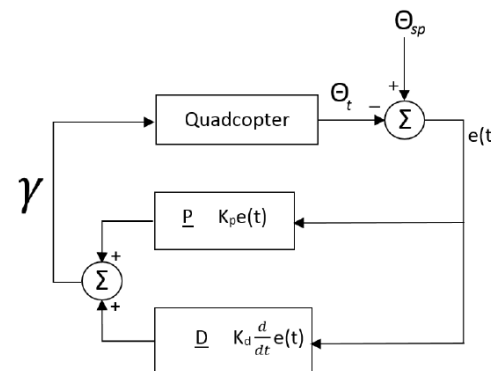


credit: norunway.com



$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \sum \frac{\mathbf{F}}{m} = \frac{1}{m} f_T \mathbf{R} \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^4 \gamma_i^2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \frac{1}{m} f_D \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\omega}_\phi \\ \dot{\omega}_\theta \\ \dot{\omega}_\psi \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}^{-1} \begin{bmatrix} \tau_\phi - (I_{zz} - I_{yy})\omega_\theta\omega_\psi \\ \tau_\theta - (I_{xx} - I_{zz})\omega_\psi\omega_\phi \\ \tau_\psi - (I_{yy} - I_{xx})\omega_\phi\omega_\theta \end{bmatrix}$$



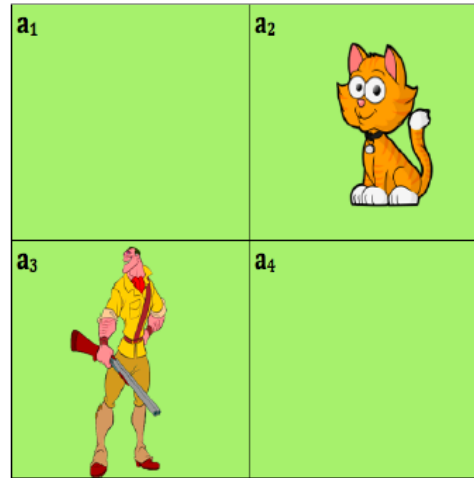
- $e(t) = \Theta_{sp} - \Theta_t$

- $\Theta_{sp} = \begin{bmatrix} 0 \\ 0 \\ \psi_o \end{bmatrix}$

- $K_p, K_d > 0$

$$PD \text{ Control} = K_p e(t) + K_d \frac{d}{dt} e(t) \rightarrow \gamma = [\gamma_1, \gamma_2, \gamma_3, \gamma_4,]$$

Drones and Target Detection



Target Detection

$$d_{a_i}^t = \begin{cases} 0 & \text{if } x_\tau \neq a_i \text{ at time } t \\ 1 & \text{if } x_\tau = a_i \text{ at time } t \end{cases}$$

Measurement Error

- β = missed detection (missing the poacher)
- α = false alarm (detecting something that's not there; a ranger, a cat, etc)

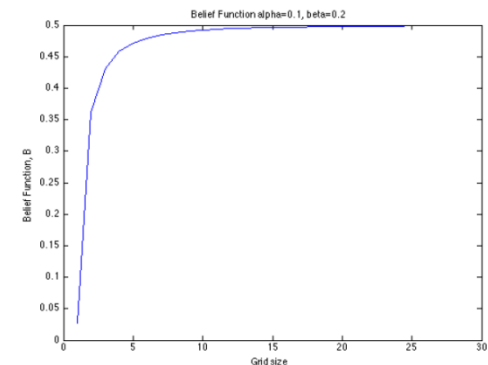
$$Bel(x_\tau) = Pr(x_\tau = a_k | D^t) = \frac{Pr(d_k^t | x_\tau = a_k, D^{t-1}) Pr(x_\tau = a_k | D^{t-1})}{Pr(d_k^t | D^{t-1})}$$

Theorem

For a uniform distribution, the belief function:

$$Pr(x_\tau \in A | D^t = \mathbf{0}) = \frac{t\beta\delta + (1 - \alpha)(|A| - t)\delta}{t\beta\delta + (1 - \alpha)(|A| - t\delta)}$$

converges to the prior belief, δ .



 **SUSTAINABLE DEVELOPMENT GOALS**

1 NO POVERTY



2 ZERO HUNGER



3 GOOD HEALTH AND WELL-BEING



4 QUALITY EDUCATION



5 GENDER EQUALITY




6 CLEAN WATER AND SANITATION



7 AFFORDABLE AND CLEAN ENERGY



8 DECENT WORK AND ECONOMIC GROWTH



9 INDUSTRY, INNOVATION AND INFRASTRUCTURE



10 REDUCED INEQUALITIES



11 SUSTAINABLE CITIES AND COMMUNITIES



12 RESPONSIBLE CONSUMPTION AND PRODUCTION



13 CLIMATE ACTION



14 LIFE BELOW WATER



15 LIFE ON LAND



16 PEACE, JUSTICE AND STRONG INSTITUTIONS



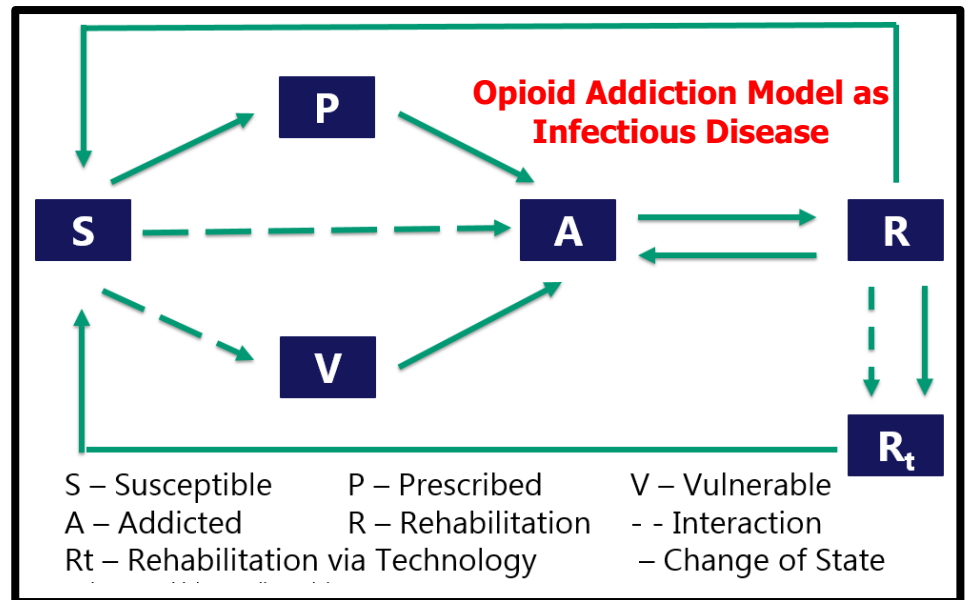
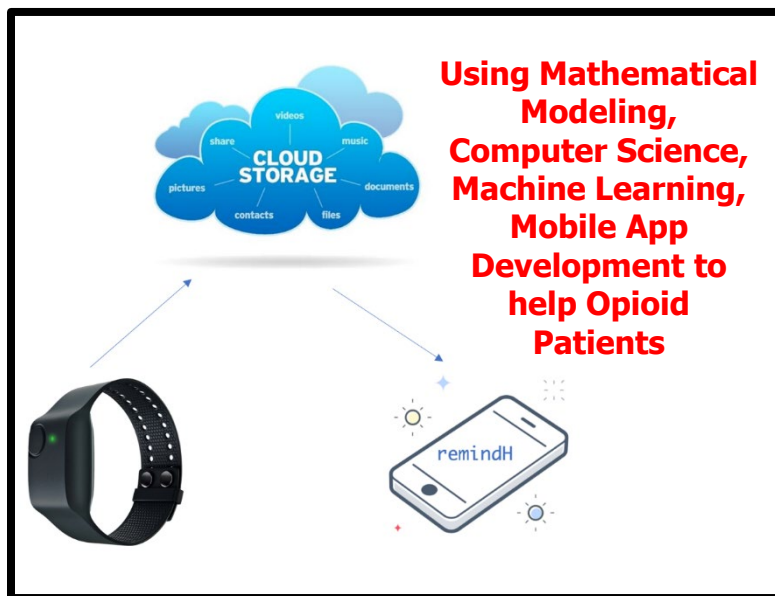
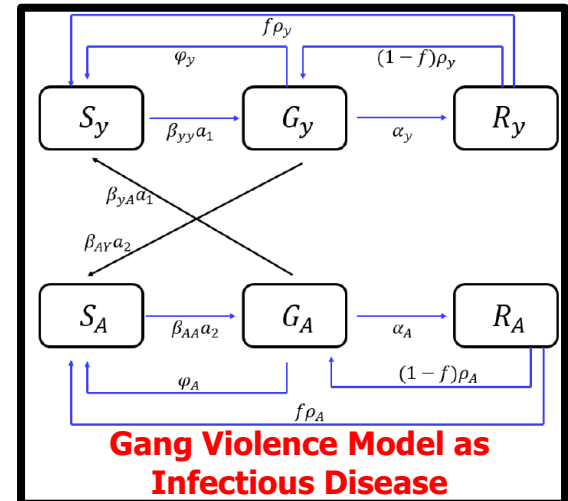
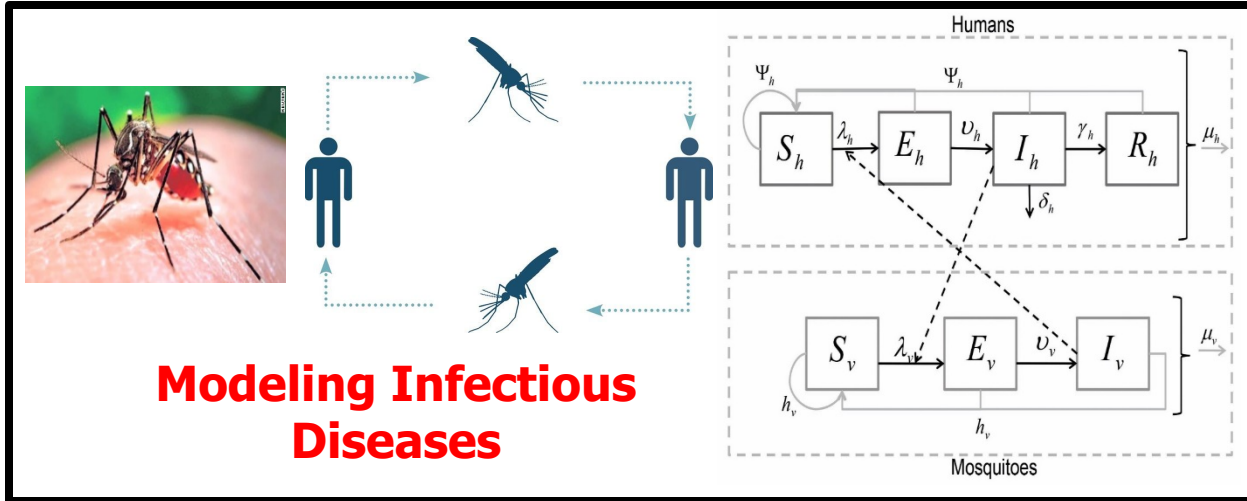
17 PARTNERSHIPS FOR THE GOALS

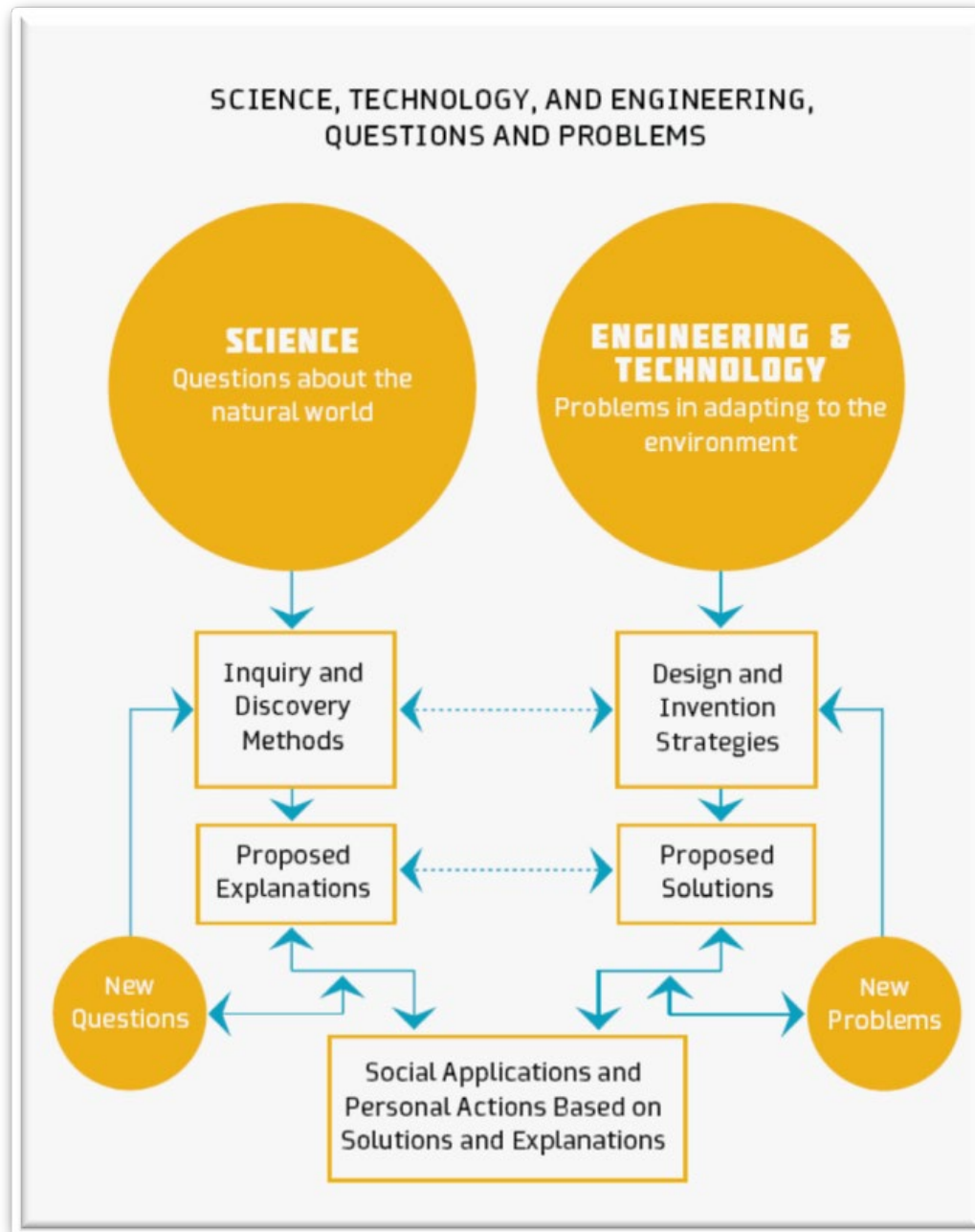



SUSTAINABLE DEVELOPMENT GOALS

Innovation to Entrepreneurship

Zika-Gangs-Opioids-Wearables





Multidisciplinary Approach to Engage Diverse Learner in Applied Mathematics Education

- As concluded by the National Research Council:
Undergraduate education will not change in a permanent way through the efforts of “Lone Rangers.” Change requires ongoing interaction among communities of people and institutions that will reinforce and drive reform.
- Research that happens across traditional STEM disciplines and at the edges of traditional disciplines.
- *Here is the problem,
find the Mathematics to solve it!*



Criminal Investigations
and Network Analysis
A DHS CENTER OF EXCELLENCE

Virtual Event

CINA Distinguished Speaker Series

Mathematics for Solving Global Challenges Involving
Criminal Activities

Date: Thursday April 30rd, 2020

Time: 12:00-1:30pm

Location: Virtual Event



Featuring:

Padhu Seshaiyer
George Mason University

Associate Dean, College of Science
Professor of Mathematical Sciences

RSVP and livestream link at
cina.gmu.edu/cinaseries

cina.gmu.edu



Contact

Padmanabhan (**Padhu**) Seshaiyer

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Linked In: <https://www.linkedin.com/in/padhuseshaiyer/>

Phone: +1 (703) 993 9787

WhatsApp: +1 (806) 239 8738

Web: <http://math.gmu.edu/~pseshaiy/outreach.html>



TEDx

<https://www.youtube.com/watch?v=Ybxnfv203k4>